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Math for Machine Learning

Linear algebra - Week 1

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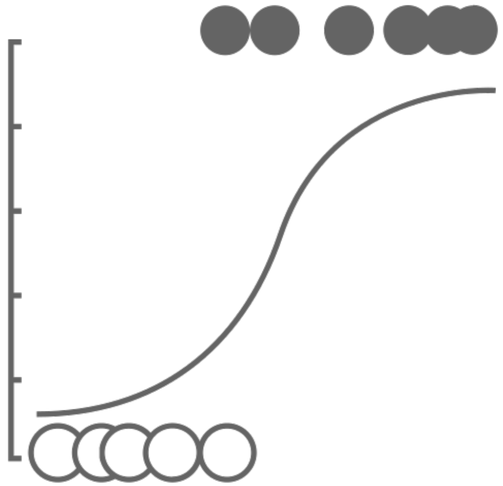
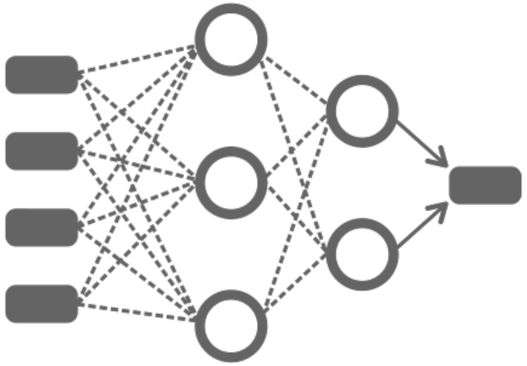
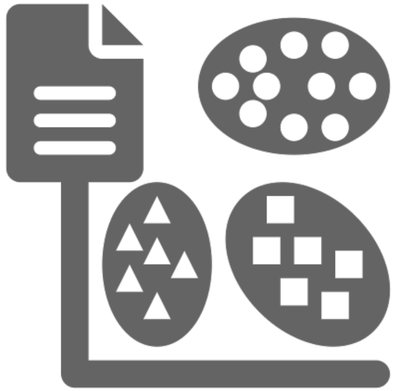
System of Linear Equations

Linear Algebra Applied I

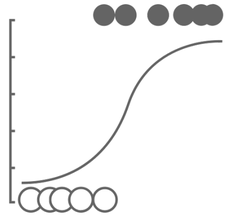
Machine Learning

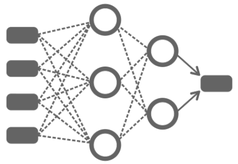


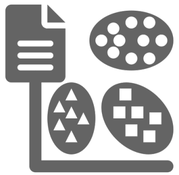




Machine Learning







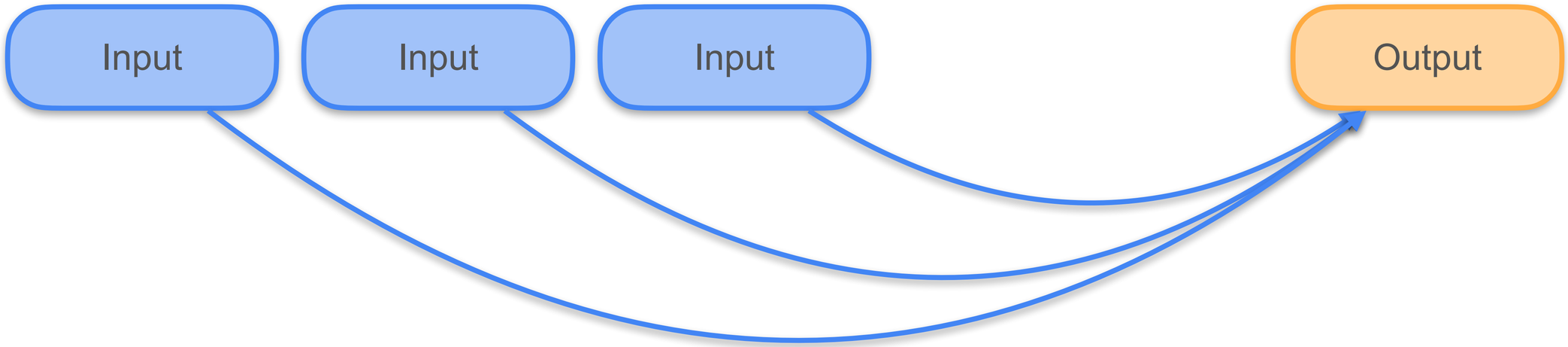
Don't worry about the math!

Don't worry about the machine learning!

Linear Algebra and Machine Learning

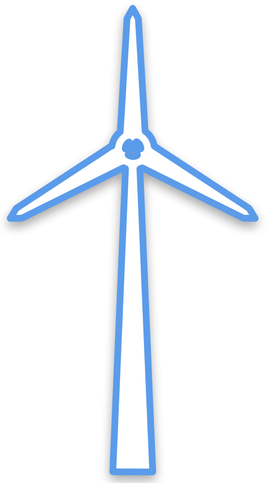
Linear Regression

Supervised Machine Learning



Linear Algebra and Machine Learning

Input



wind speed

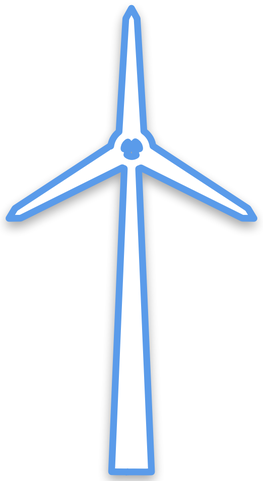
Output

power output

Linear Algebra and Machine Learning

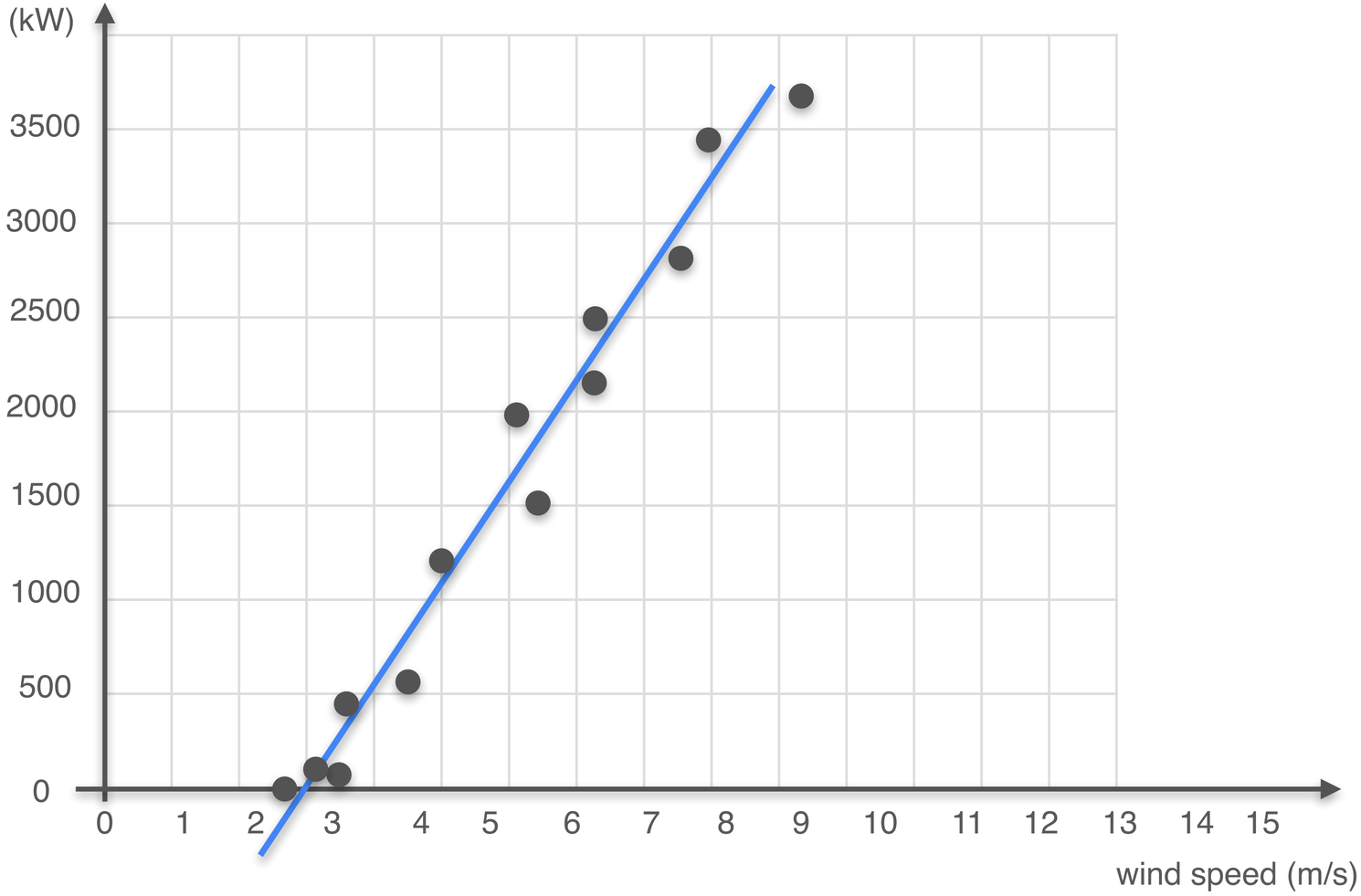
Input

Output



wind speed

power output



Linear Algebra and Machine Learning

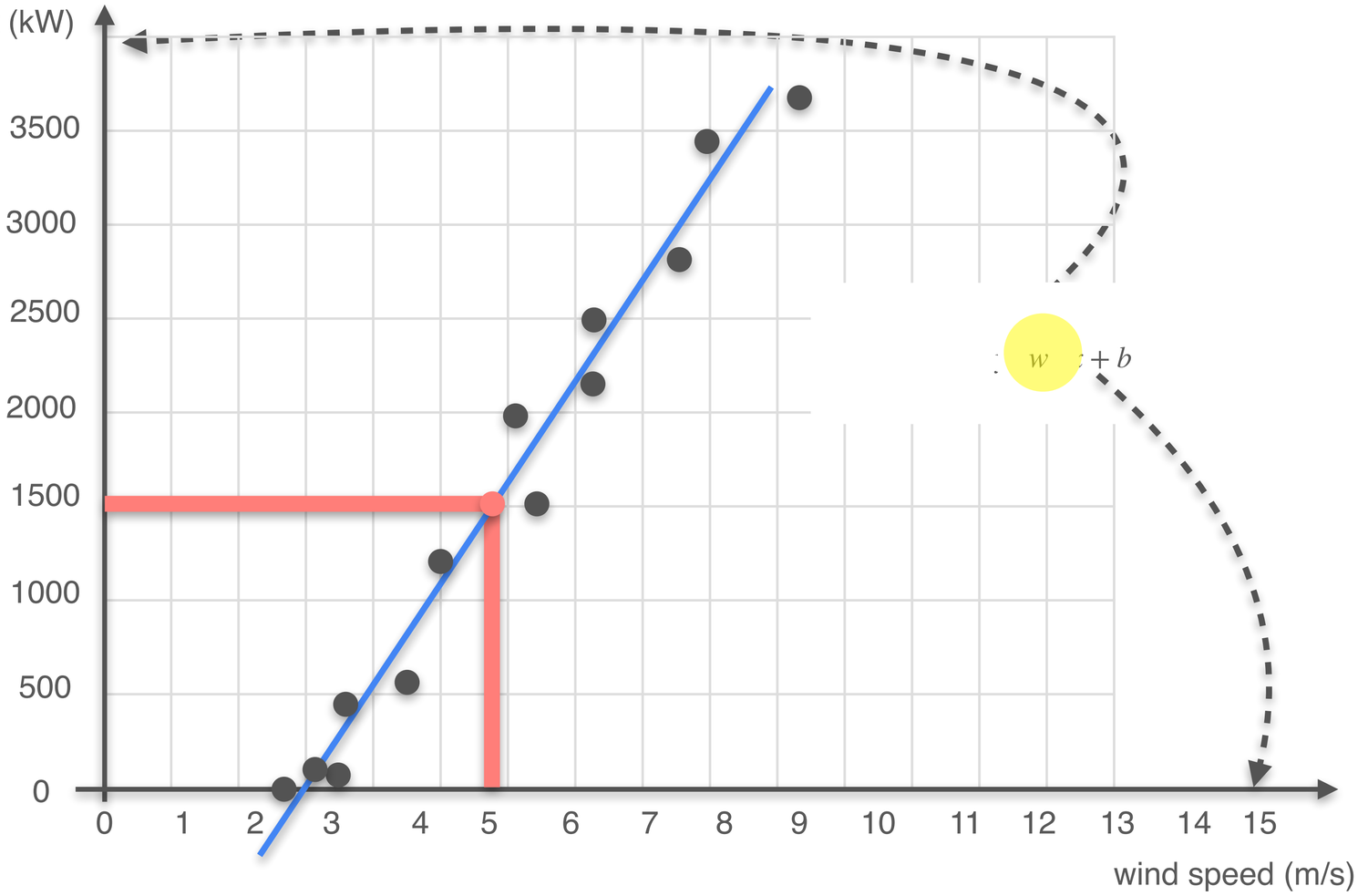
Input

Output

$m \times$ wind speed $+ b$ = power output

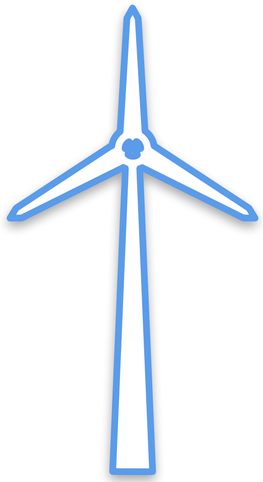
5m/s

1500kW



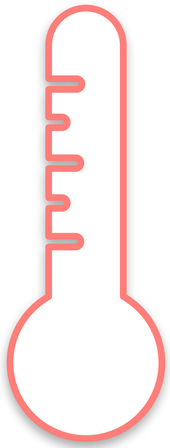
Linear Algebra and Machine Learning

Input



wind speed

Input

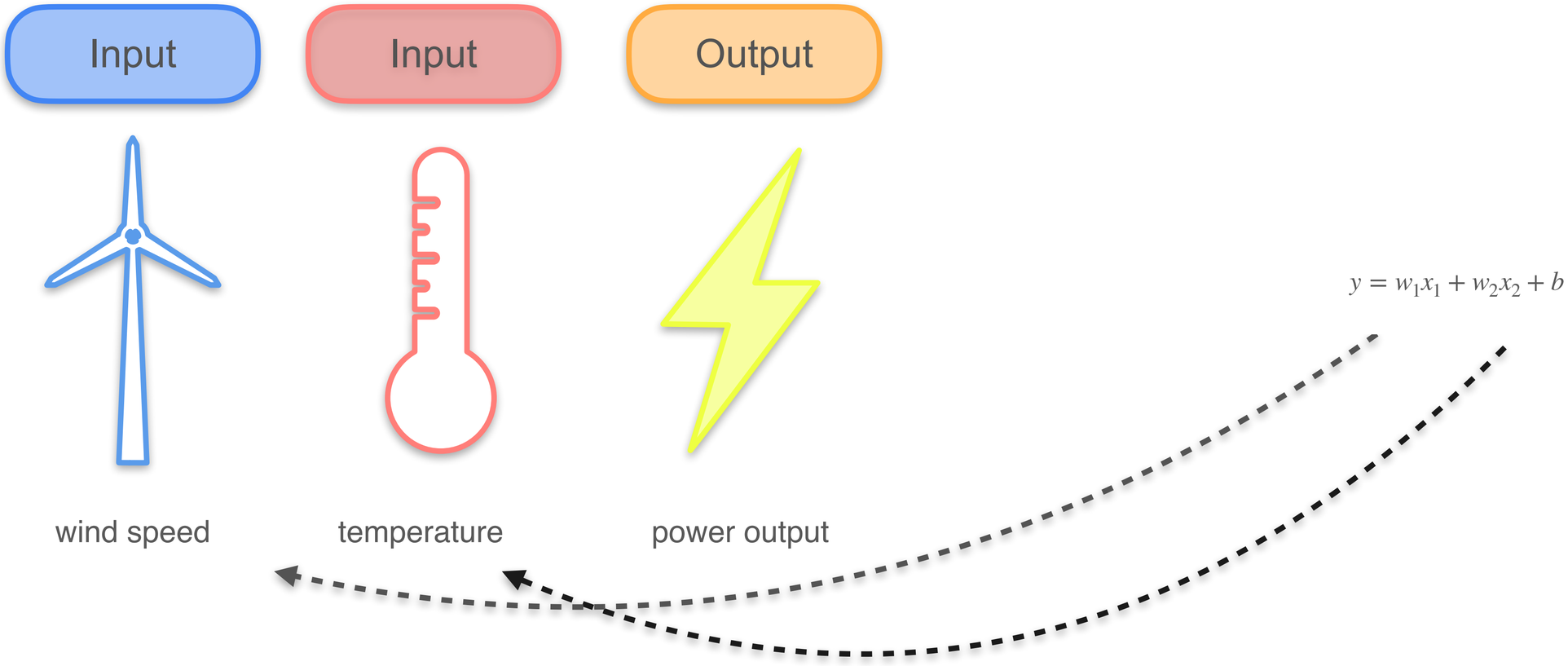


temperature

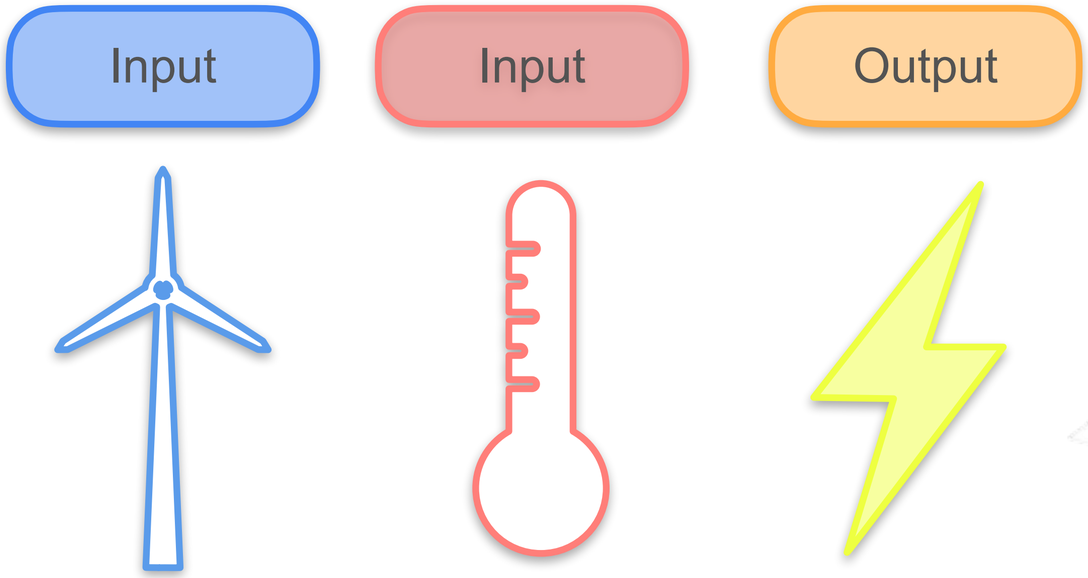
Output

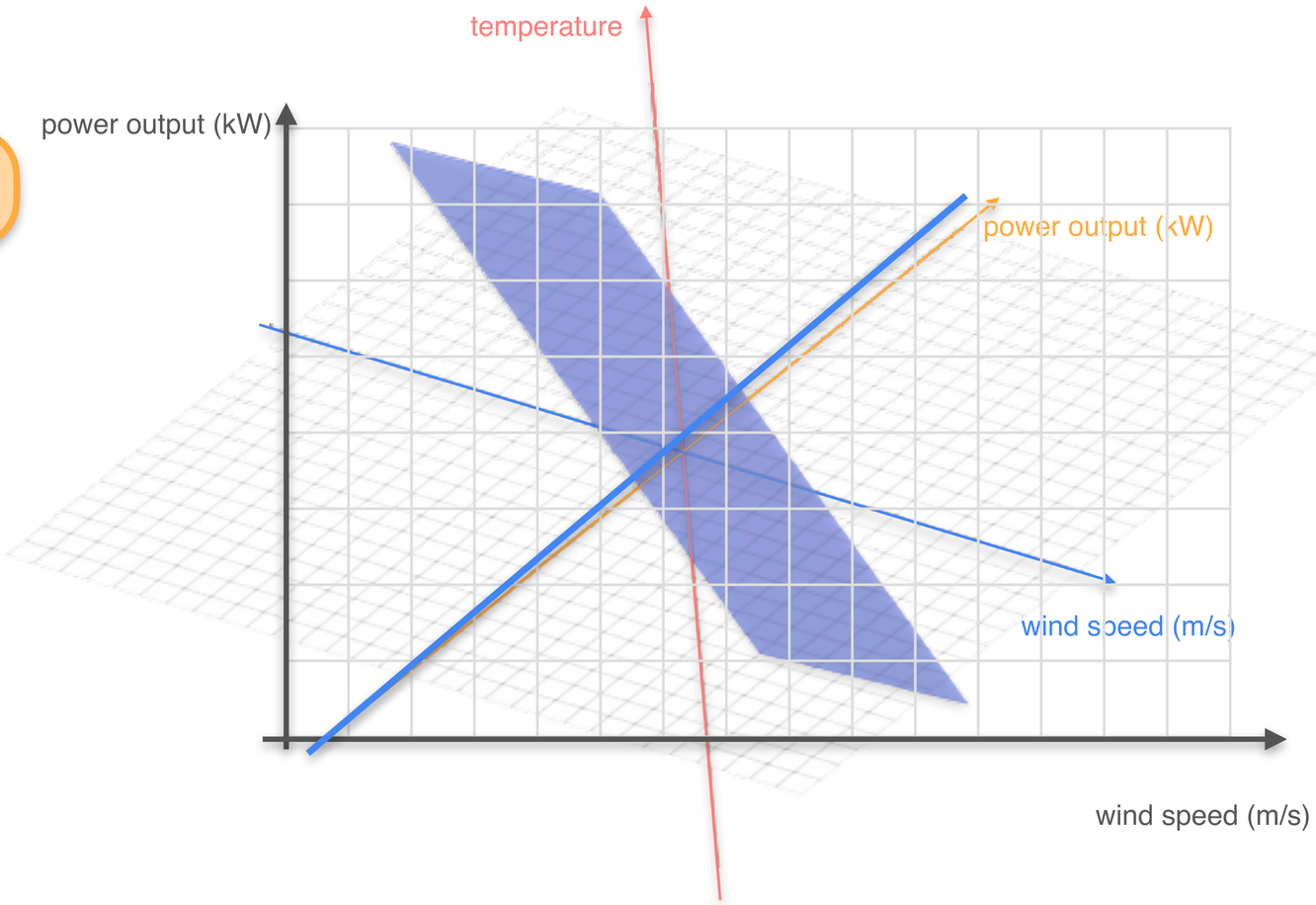
power output

Linear Algebra and Machine Learning

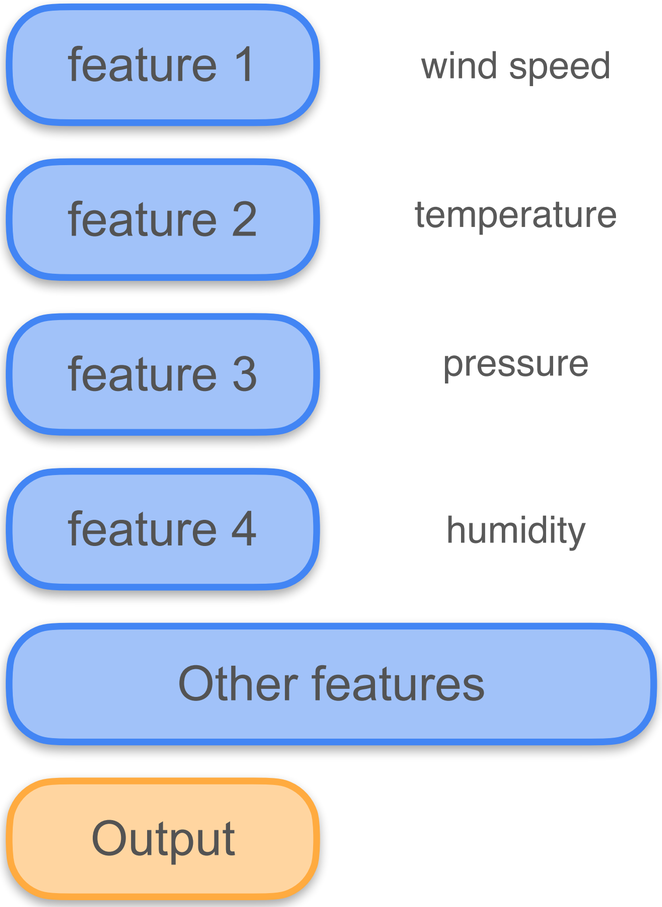


Linear Algebra and Machine Learning

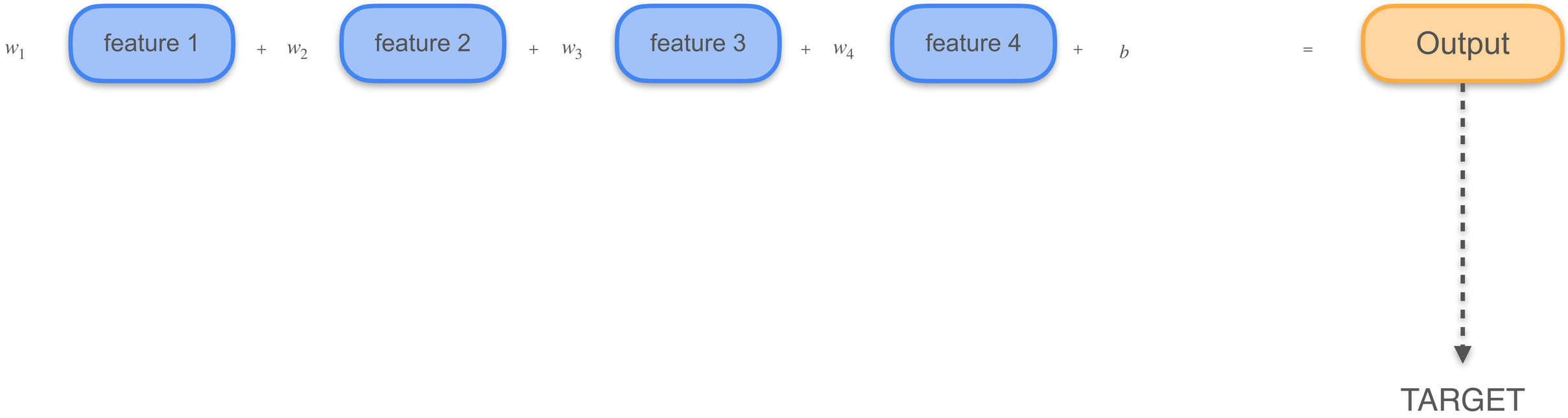




Linear Algebra and Machine Learning

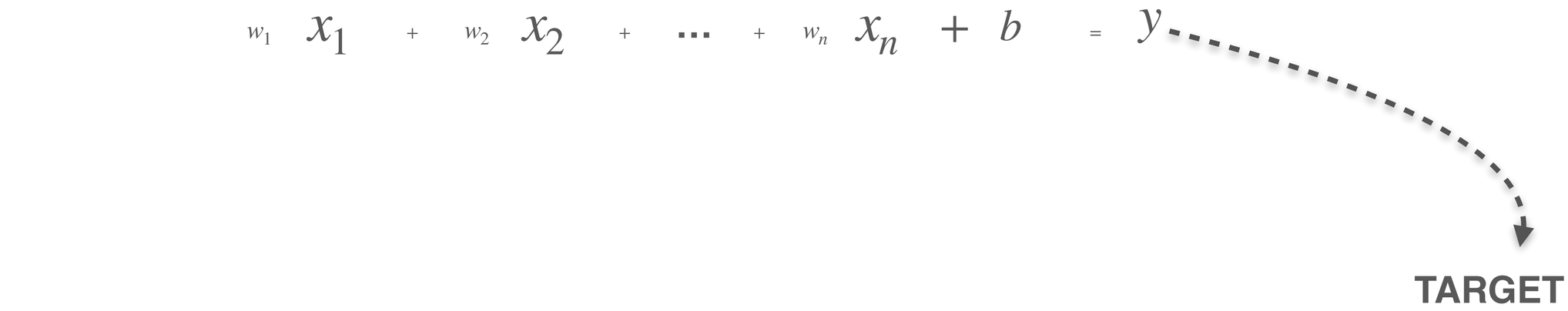


Linear Algebra and Machine Learning



Linear Algebra and Machine Learning

$$w_1 x_1 + w_2 x_2 + \cdots + w_n x_n + b = y$$



Linear Algebra and Machine Learning

$$
\begin{aligned}
w_1\ x_1^{(1)} + w_2\ x_2^{(1)} + \cdots + w_n\ x_n^{(1)} + b &= y^{(1)} \\
w_1\ x_1^{(2)} + w_2\ x_2^{(2)} + \cdots + w_n\ x_n^{(2)} + b &= y^{(2)} \\
w_1\ x_1^{(3)} + w_2\ x_2^{(3)} + \cdots + w_n\ x_n^{(3)} + b &= y^{(3)} \\
&\vdots \\
w_1\ x_1^{(m)} + w_2\ x_2^{(m)} + \cdots + w_n\ x_n^{(m)} + b &= y^{(m)}
\end{aligned}
$$

System of Linear Equations

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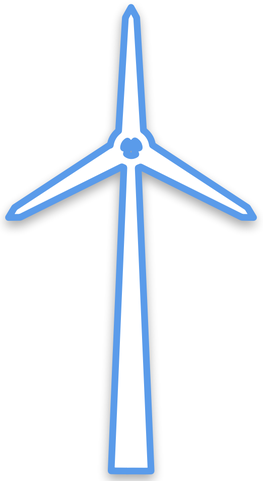
System of Linear Equations

Linear Algebra Applied II

Linear Algebra and Machine Learning

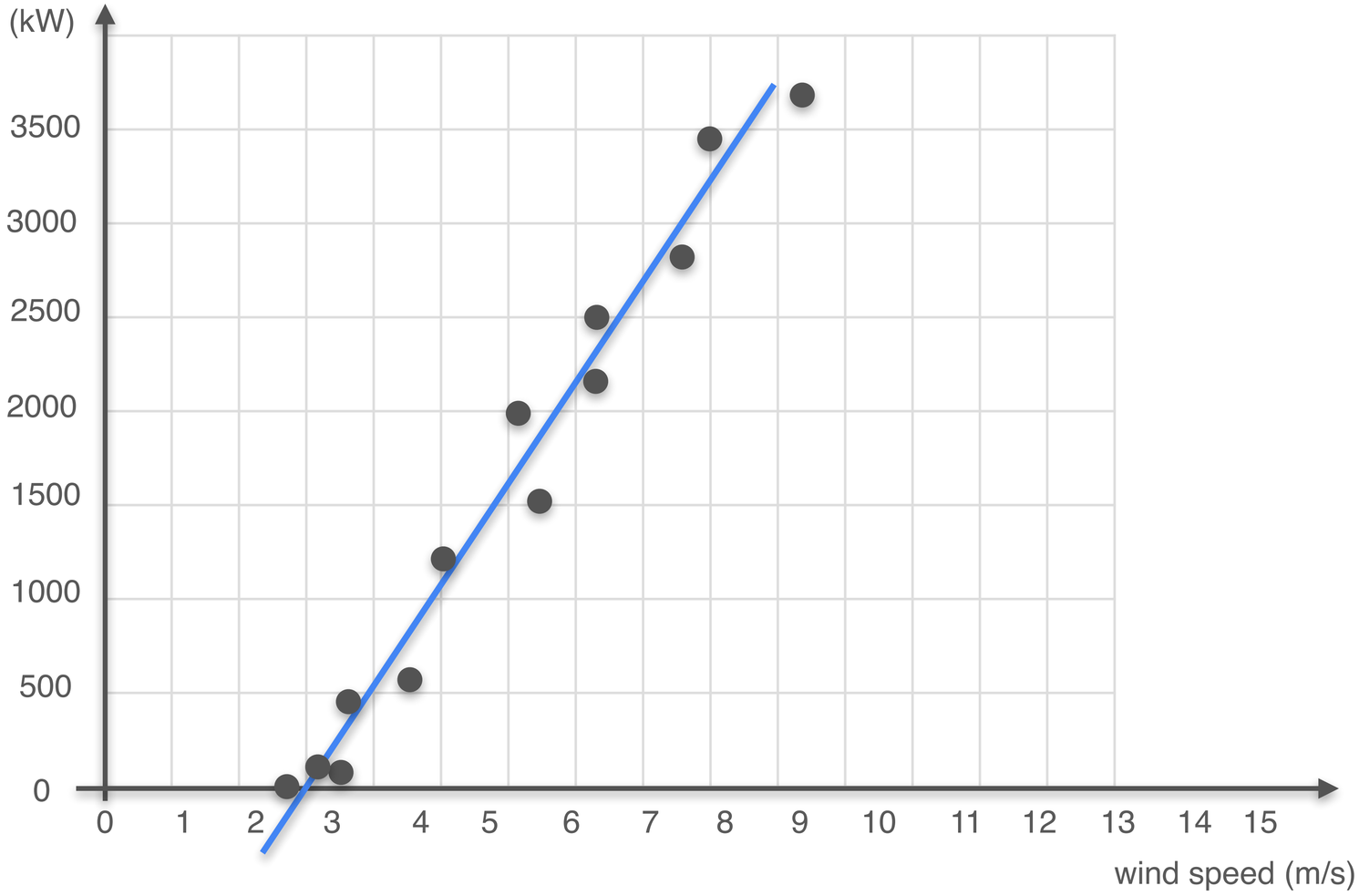
Input

Output

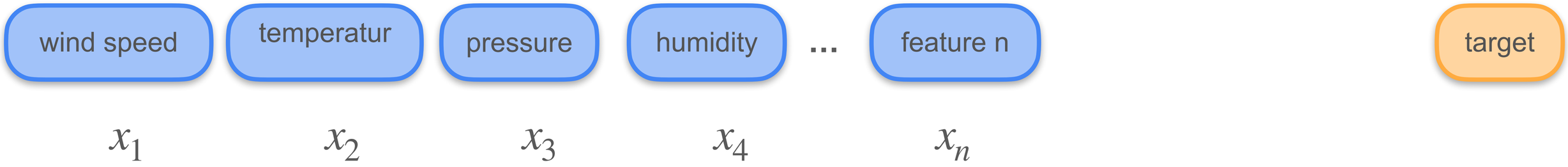


wind speed

electrical power output



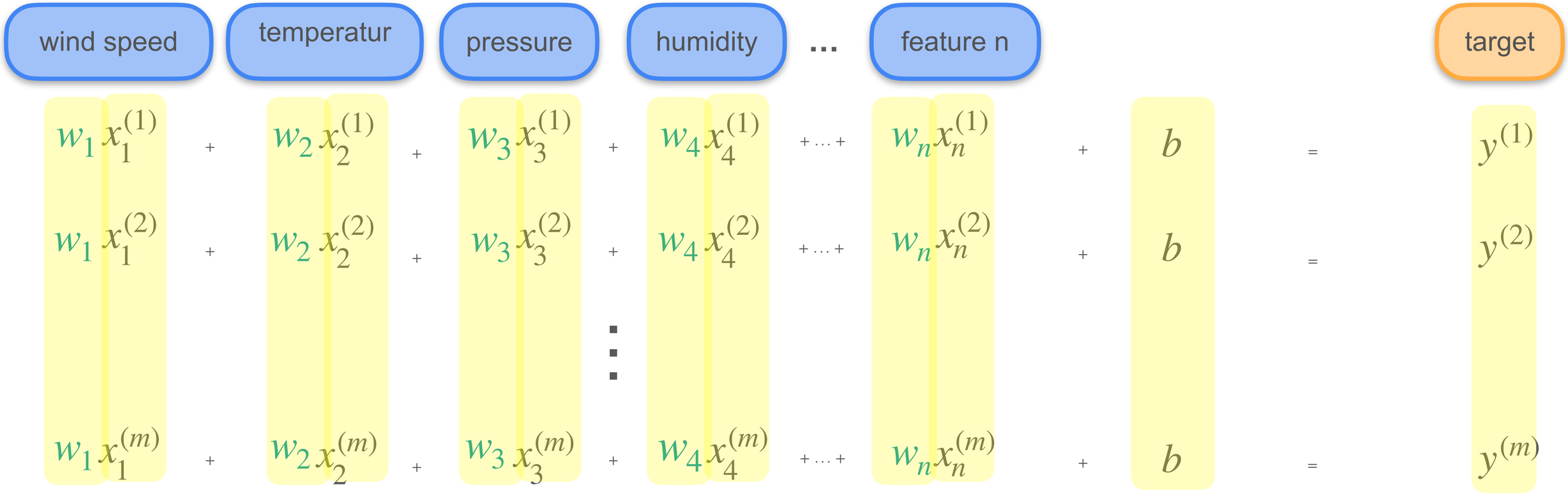
Linear Algebra and Machine Learning



Linear Algebra and Machine Learning

wind speed	temperatur	pressure	humidity	...	feature n			target
$w_1 x_1^{(1)}$	$+ \; w_2 x_2^{(1)}$	$+ \; w_3 x_3^{(1)}$	$+ \; w_4 x_4^{(1)}$	$+ \ldots +$	$w_n x_n^{(1)}$	$+ \; b$	$=$	$y^{(1)}$
$w_1 x_1^{(2)}$	$+ \; w_2 x_2^{(2)}$	$+ \; w_3 x_3^{(2)}$	$+ \; w_4 x_4^{(2)}$	$+ \ldots +$	$w_n x_n^{(2)}$	$+ \; b$	$=$	$y^{(2)}$
$\vdots$								
$w_1 x_1^{(m)}$	$+ \; w_2 x_2^{(m)}$	$+ \; w_3 x_3^{(m)}$	$+ \; w_4 x_4^{(m)}$	$+ \ldots +$	$w_n x_n^{(m)}$	$+ \; b$	$=$	$y^{(m)}$

Linear Algebra and Machine Learning



Linear Algebra and Machine Learning

$$w \cdot X + b = Y$$

$$\begin{bmatrix} w_1 & w_2 & w_3 & w_4 & \dots & w_n \end{bmatrix}$$

vector

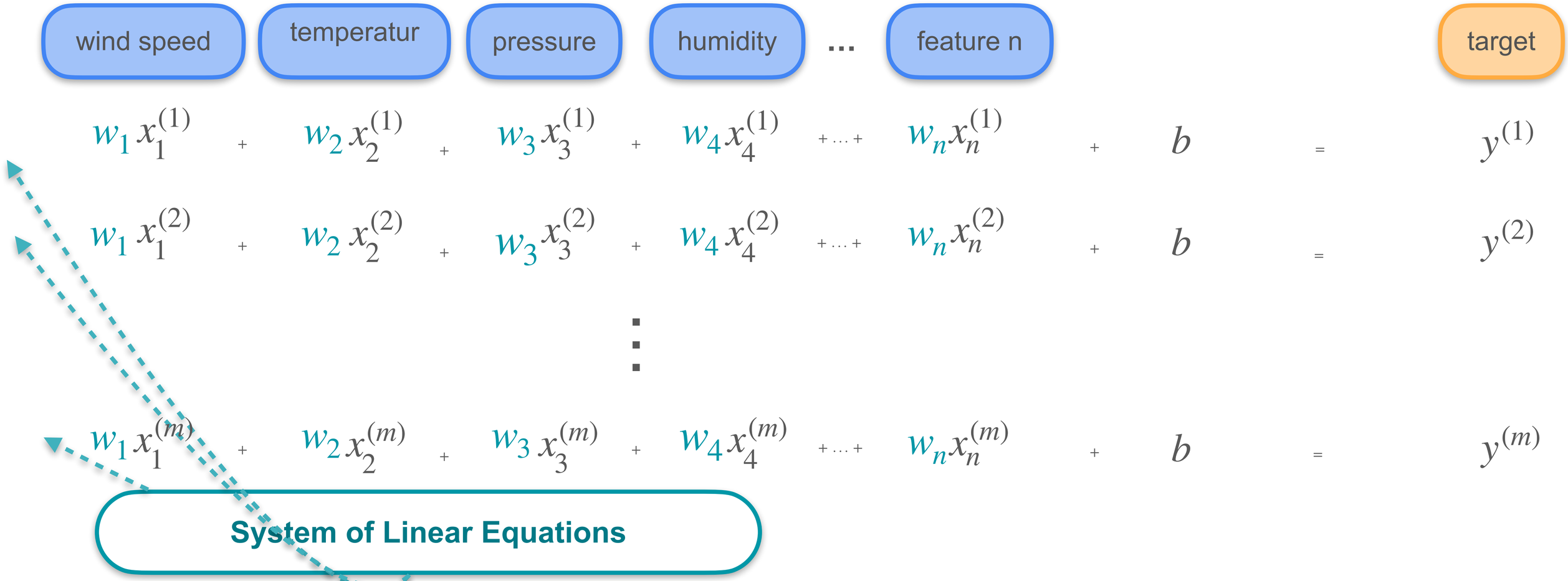
$$\begin{bmatrix} x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & x_4^{(1)} & \dots & x_n^{(1)} \\ x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & x_4^{(2)} & \dots & x_n^{(2)} \\ \vdots & & & & & \\ x_1^{(m)} & x_2^{(m)} & x_3^{(m)} & x_4^{(m)} & \dots & x_n^{(m)} \end{bmatrix}$$

matrix

$$\begin{bmatrix} y^{(1)} & y^{(2)} & \dots & y^{(m)} \end{bmatrix}$$

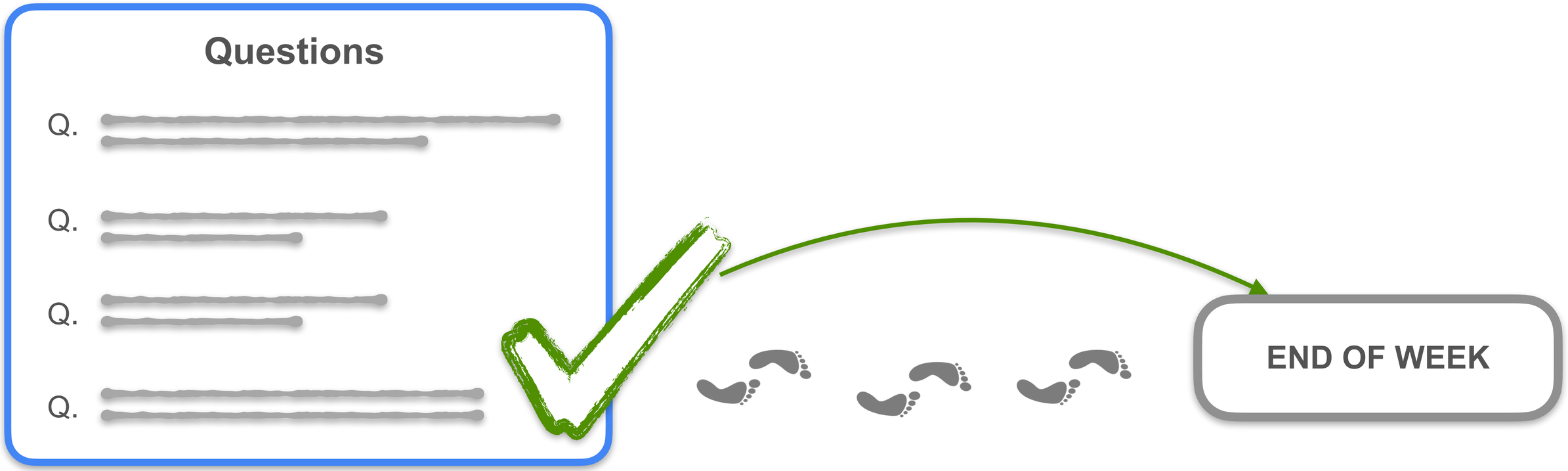
vector

Linear Algebra and Machine Learning



Plan for the Week

Common vector and matrix operations



Plan for the Week

Systems of Linear Equations

Representing systems as vectors and matrices

Computing the determinant of matrices

Check your Knowledge

Linear Algebra	Your algebra score added to your calculus score minus your probability score was 6
Calculus	Your algebra score minus your calculus score plus double your probability score was 4.
Probability & Statistics	Four times your algebra score minus double your calculus score added to your probability score was 10

Represent these statements as a system of linear equations.

Check your Knowledge

a	Linear Algebra	Your algebra score added to your calculus score minus your probability score was 6	$a + c - p = 6$
c	Calculus	Your algebra score minus your calculus score plus double your probability score was 4.	$a - c + 2p = 4$
p	Probability & Statistics	Four times your algebra score minus double your calculus score added to your probability score was 10	$4a - 2c + p = 10$

Represent these statements as a system of linear equations.

Check your Knowledge

What are the weights, w? a, c, p

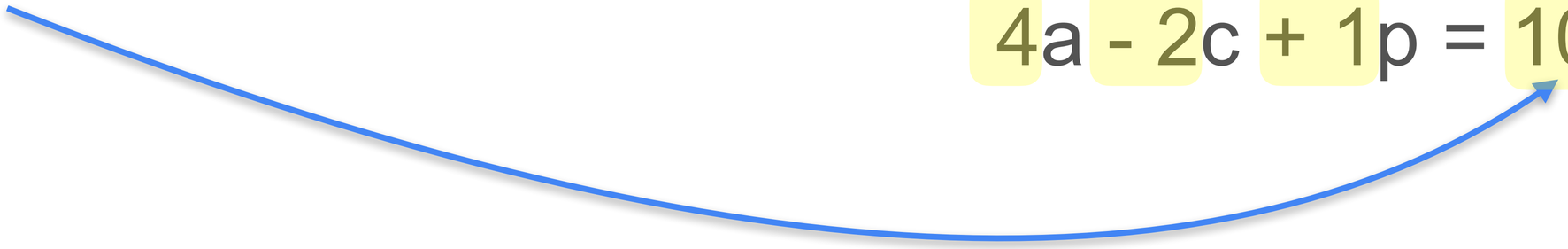
What are the features, x?

The targets, y? 6, 4, 10

$1a + 1c - 1p = 6$

$1a - 1c + 2p = 4$

$4a - 2c + 1p = 10$



Check your Knowledge

Is this system singular or non-singular?

Can you solve this system of equations?

Can you represent this system as a matrix and a vector?

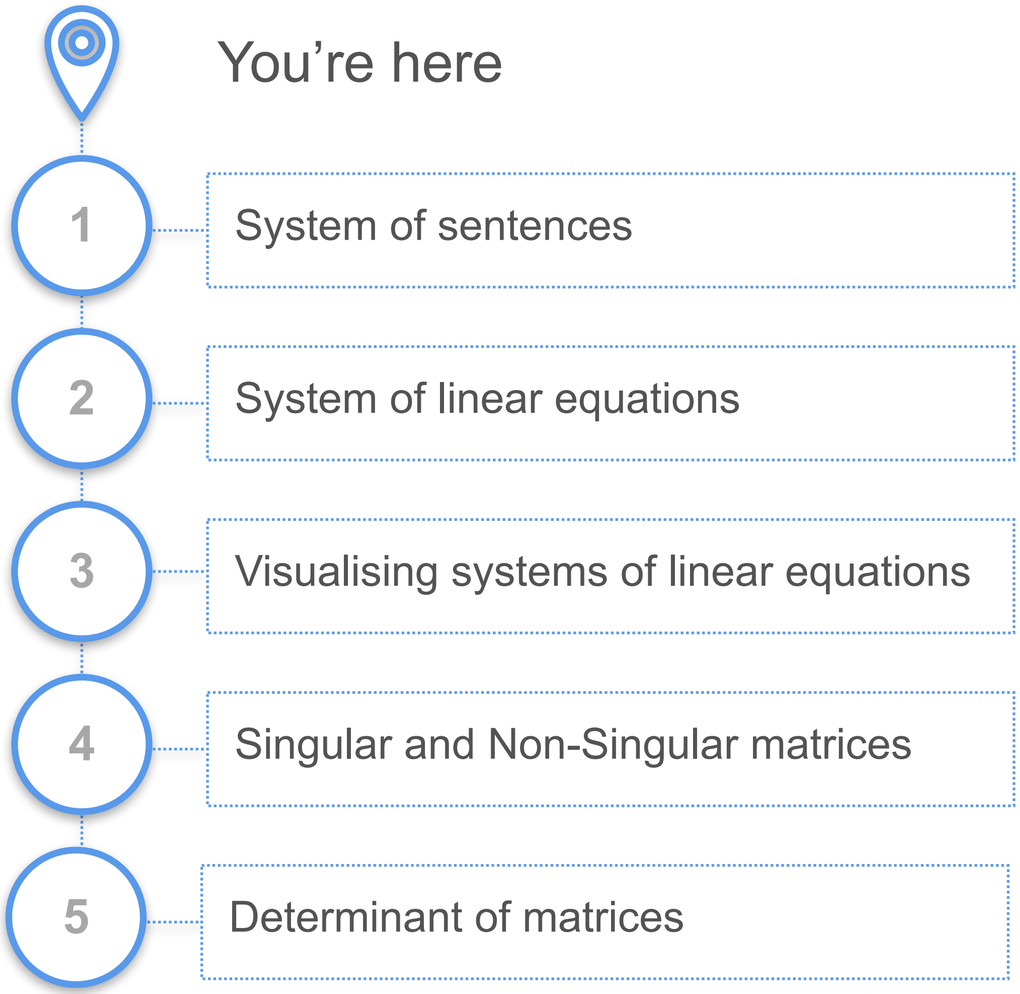
Can you calculate the determinant of that matrix?

$$a + c - p = 6$$

$$a - c + 2p = 4$$

$$4a - 2c + p = 10$$

What to expect



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System of Linear Equations

System of sentences

Systems of sentences

System 1

The dog is **black**
The cat is **orange**

Complete

Non-singular

System 2

The dog is **black**
The dog is **black**

Redundant

Singular

System 3

The dog is **black**
The dog is **white**

Contradictory

Singular

Systems of sentences

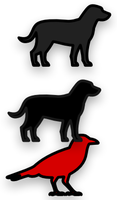
System 1

The dog is **black**
The cat is **orange**
The bird is **red**

Complete

Non-singular

System 2



The dog is **black**
The dog is **black**
The bird is **red**

Redundant

Singular

System 3

The dog is **black**
The dog is **black**
The dog is **black**

Redundant

Singular

System 4

The dog is **black**
The dog is white
The bird is **red**

Contradictory

Singular

Quiz: Systems of sentences

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.

Problem 1:

What color is the bird?

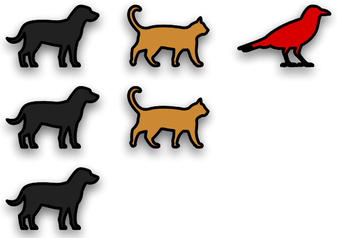
Problem 2:

Is this system singular or non-singular?

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



Solution 1:

The bird is red. 

Solution 2:

It is non-singular. 

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System of Linear Equations

System of equations

Sentences → Equations

Sentences

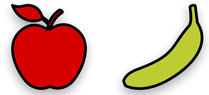
Between the dog and the cat, one is black.

Sentences with numbers

The price of an apple and a banana is $10.

Equations

a + b = 10



Quiz: Systems of equations 1

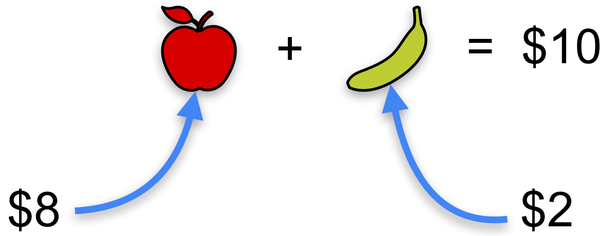
You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost $10.
- **Day 2:** You bought an apple and two bananas and they cost $12.

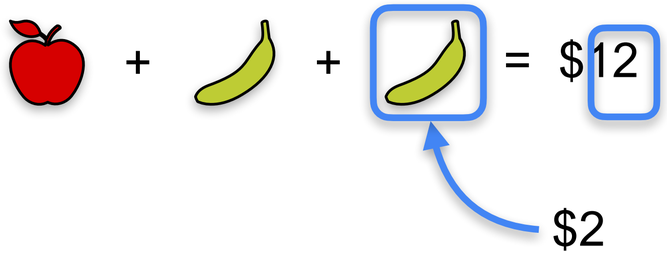
Question: How much does each fruit cost?

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost $10.



- **Day 2:** You bought an apple and two bananas and they cost $12.



- **Solution:** An apple costs $8, a banana costs $2.

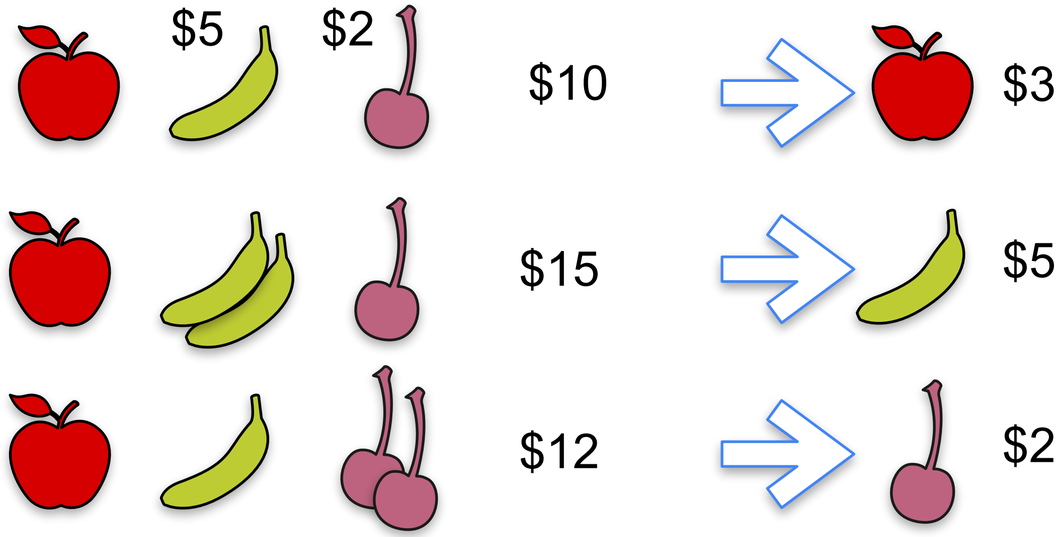
Quiz: Systems of equations 2

Problem 1: You're trying to figure out the price of apples, bananas, and cherries at the store. You go three days in a row, and bring this information.

- **Day 1:** You bought an apple, a banana, and a cherry, and paid $10.
- **Day 2:** You bought an apple, two bananas, and a cherry, and paid $15.
- **Day 3:** You bought an apple, a banana, and two cherries, and paid $12.

How much does each fruit cost?

Solution: Systems of equations 2



System of equations 1

$$a + b + c = 10$$
$$a + 2b + c = 15$$
$$a + b + 2c = 12$$

Solution

$$a = 3$$
$$b = 5$$
$$c = 2$$

Quiz: Systems of equations 3

You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost $10.
- **Day 2:** You bought two apples and two bananas and they cost $20.

Question: How much does each fruit cost?

Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost $10.

- **Day 2:** You bought two apples and two bananas and they cost $20.

Same thing!!!

8	2
5	5
8.3	1.7
0	10

Infinitely many solutions!

Quiz: Systems of equations 4

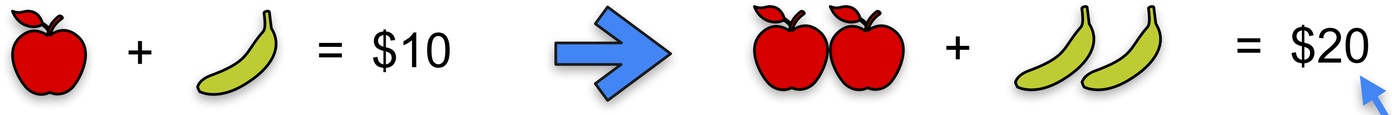
You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost $10.
- **Day 2:** You bought two apples and two bananas and they cost $24.

Question: How much does each fruit cost?

Solution: Systems of equations 4

- **Day 1:** You bought an apple and a banana and they cost $10.



- **Day 2:** You bought two apples and two bananas and they cost $24.

Contradiction!

No solutions!

Systems of equations

System 1

$a + b = 10$

$a + 2b = 12$

Unique solution:

$a = 8$

$b = 2$

Complete

Non-singular

System 2

$a + b = 10$

$2a + 2b = 20$

Infinite solutions

$a = 8, 7, 6, \ldots$

$b = 2, 3, 4$

Redundant

Singular

System 3

$a + b = 10$

$2a + 2b = 24$

No solution

Contradictory

Singular

Quiz: More systems of equations

System 1

$$a + b + c = 10$$
$$a + b + 2c = 15$$
$$a + b + 3c = 20$$

System 2

$$a + b + c = 10$$
$$a + b + 2c = 15$$
$$a + b + 3c = 18$$

System 3

$$a + b + c = 10$$
$$2a + 2b + 2c = 20$$
$$3a + 3b + 3c = 30$$

Solutions: More systems of equations

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

Infinitely many sols.

$c = 5$

$a + b = 5$

(0,5,5), (1,4,5), (2,3,5), …

System 3

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

No solutions

From 1st and 2nd:
$c = 5$
From 2nd and 3rd:
$c = 3$

System 4

$a + b + c = 10$
$2a + 2b + 2c = 20$
$3a + 3b + 3c = 30$

Infinitely many solutions

Any 3 numbers that add to 10 work.
(0,0,10), (2,7,1), …

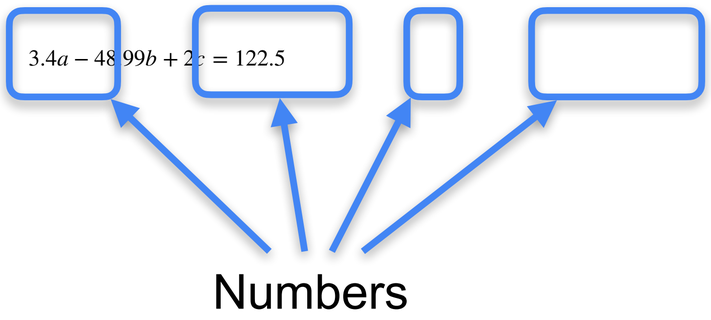
What is a linear equation?

Linear

$a + b = 10$

$2a + 3b = 15$

$3.4a - 48.99b + 2c = 122.5$



Non-linear

$a^2 + b^2 = 10$

$\sin(a) + b^5 = 15$

$2^a - 3^b = 0$

$ab^2 + \frac{b}{a} - \frac{3}{b} - \log(c) = 4^a$

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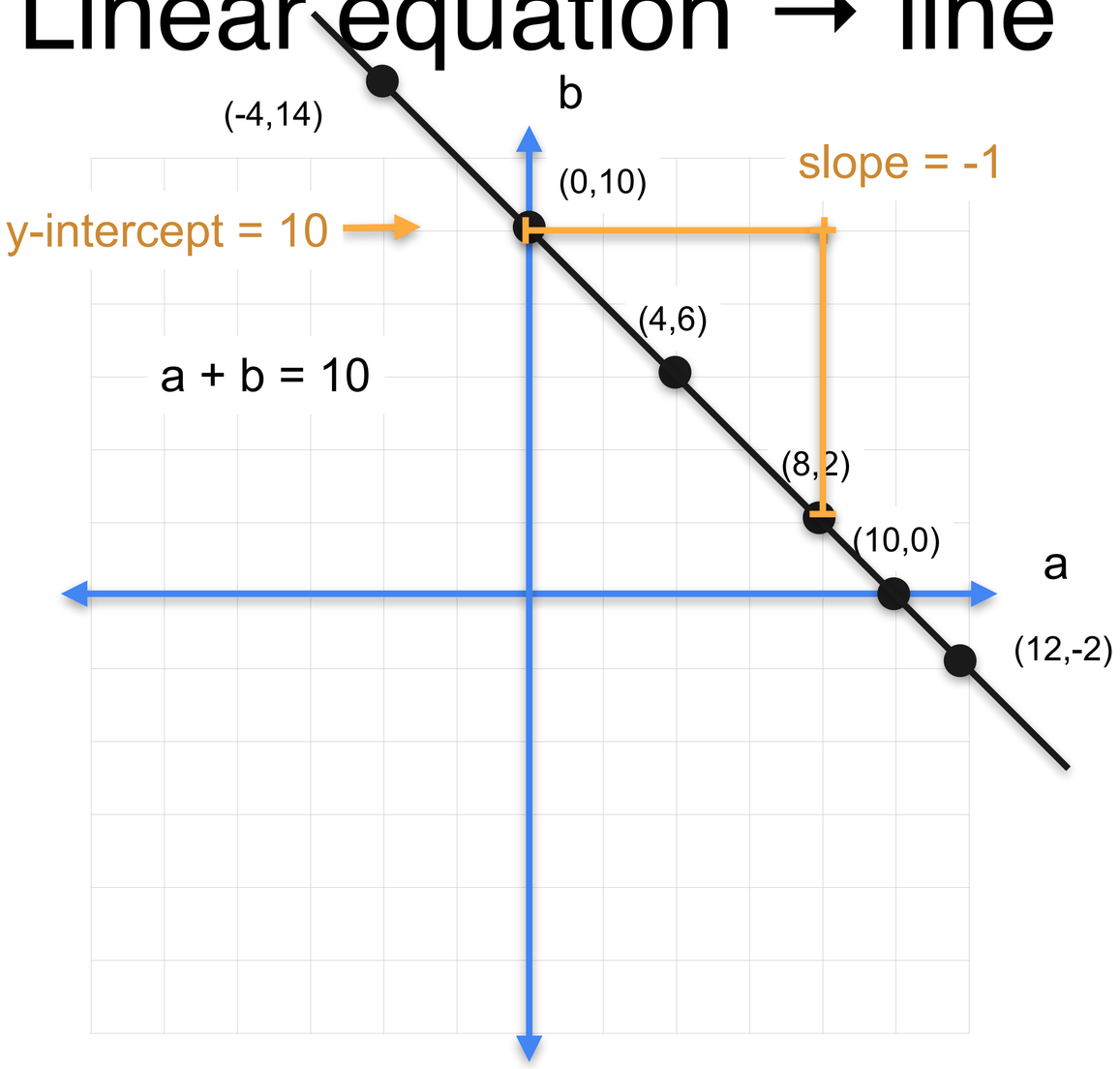
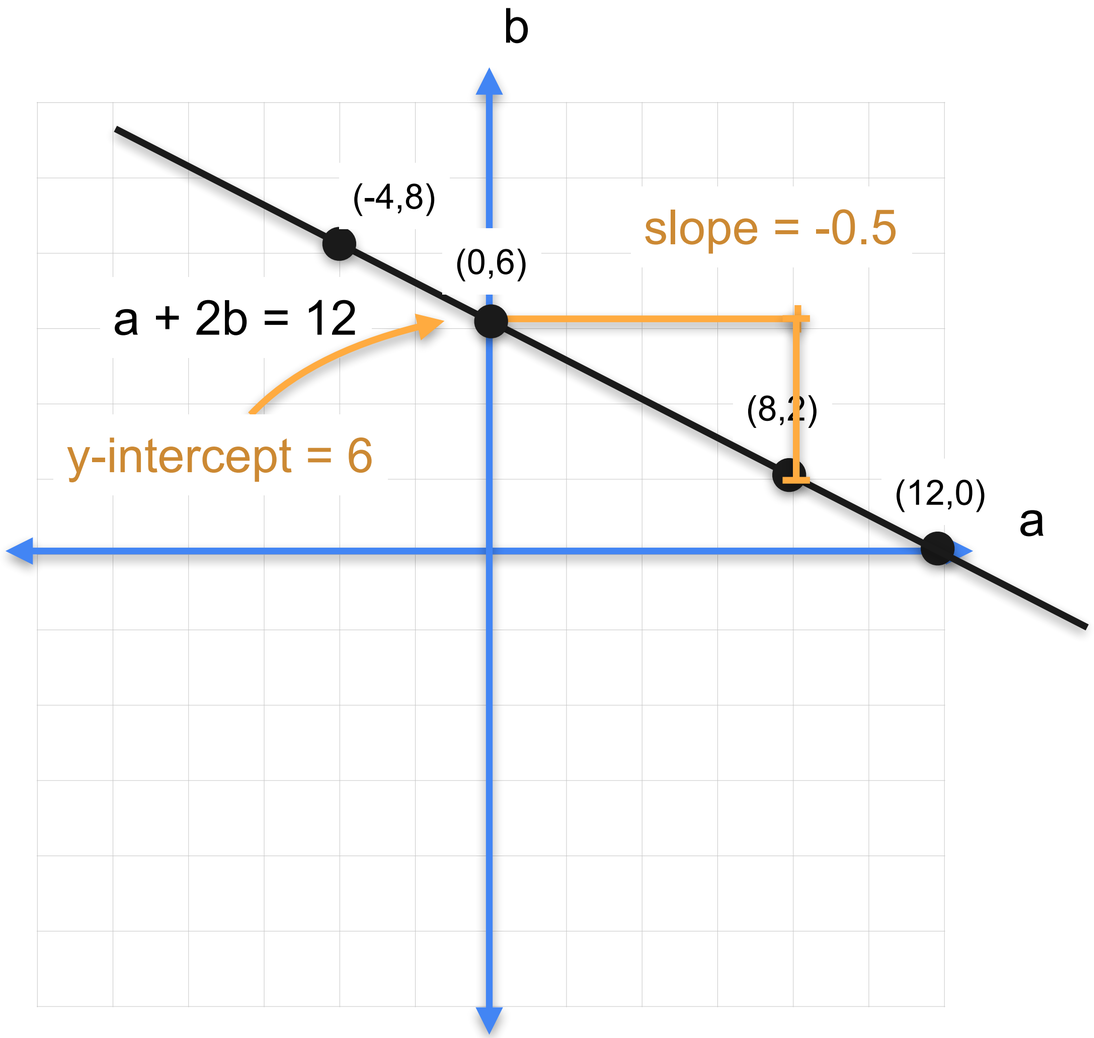
System of Linear Equations

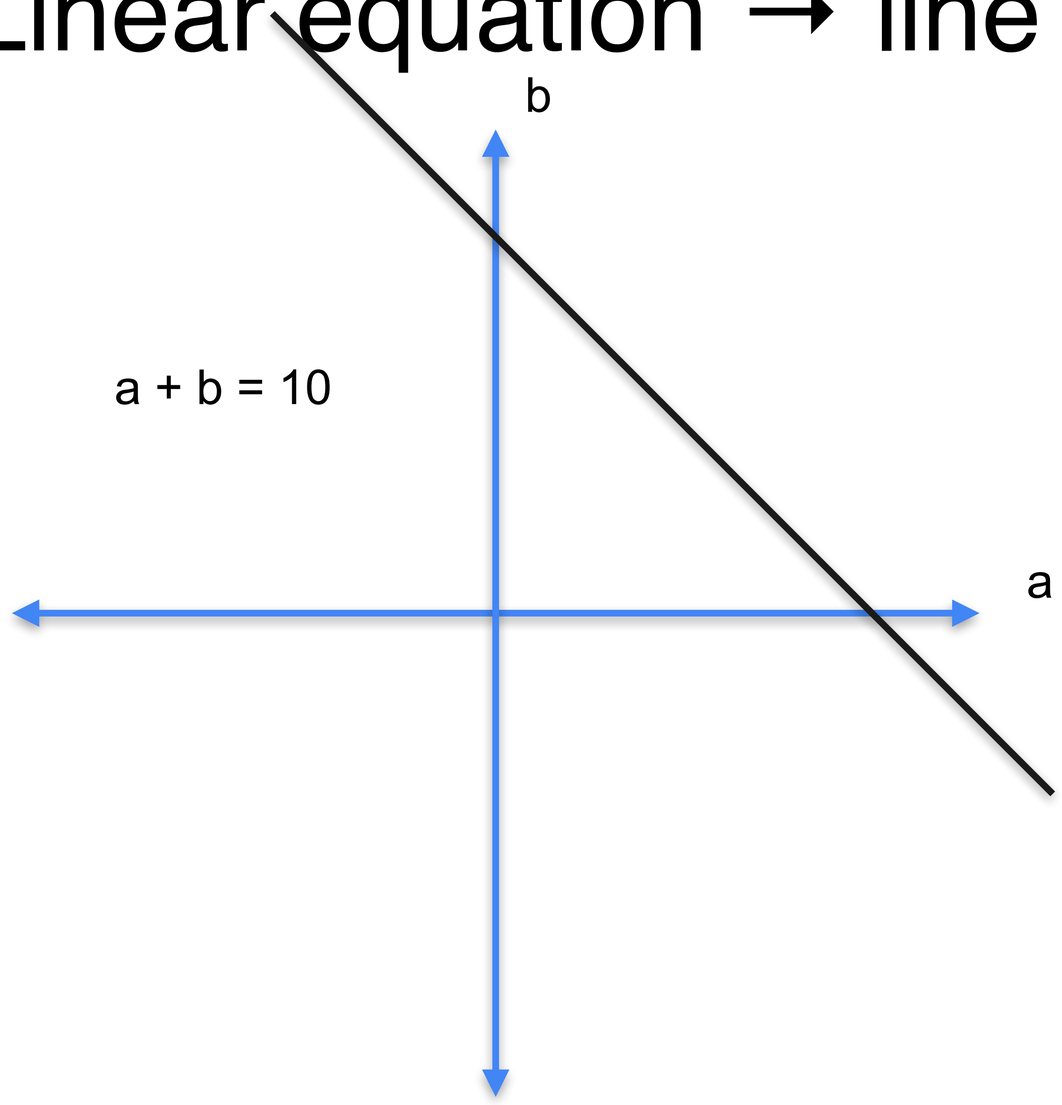
System of equations as lines and planes

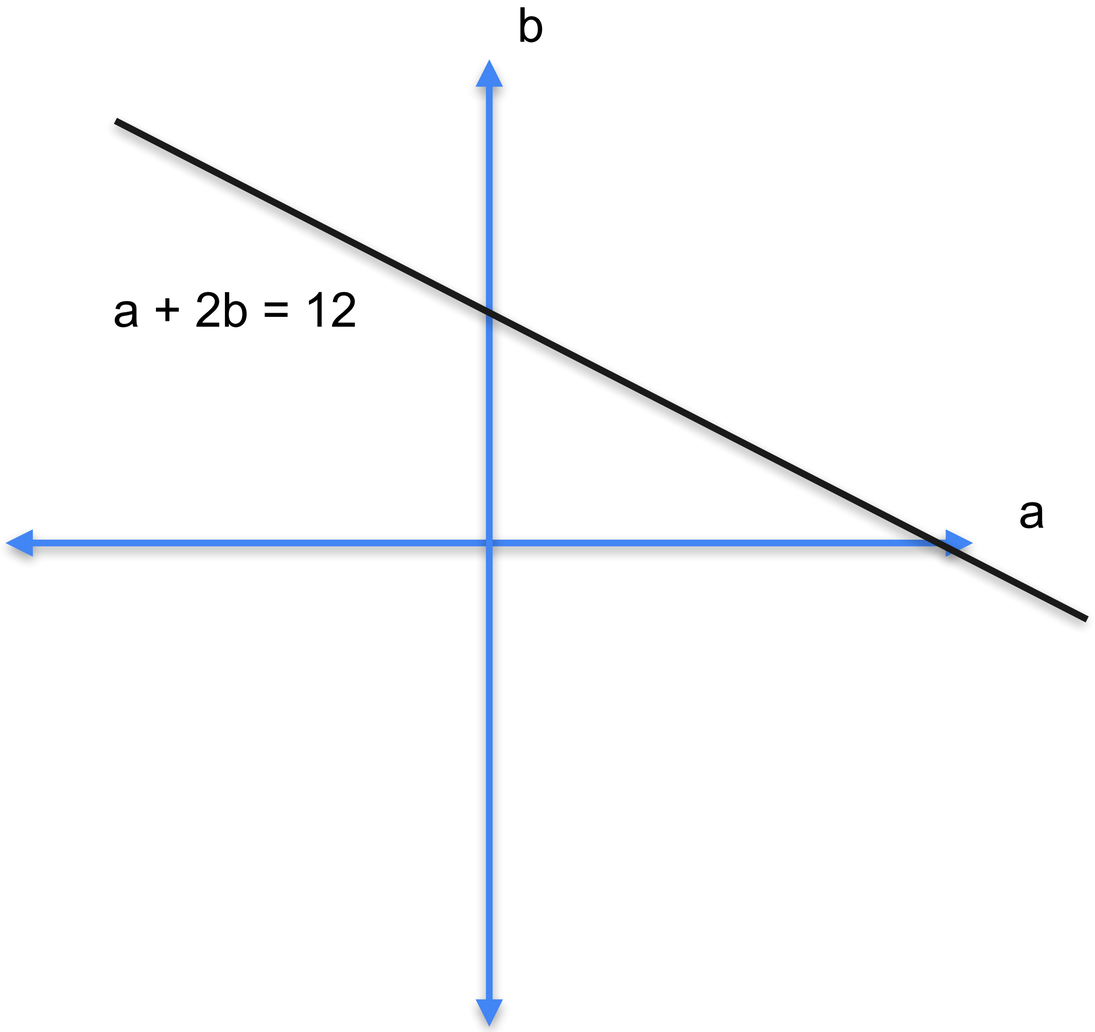
Linear equation → line



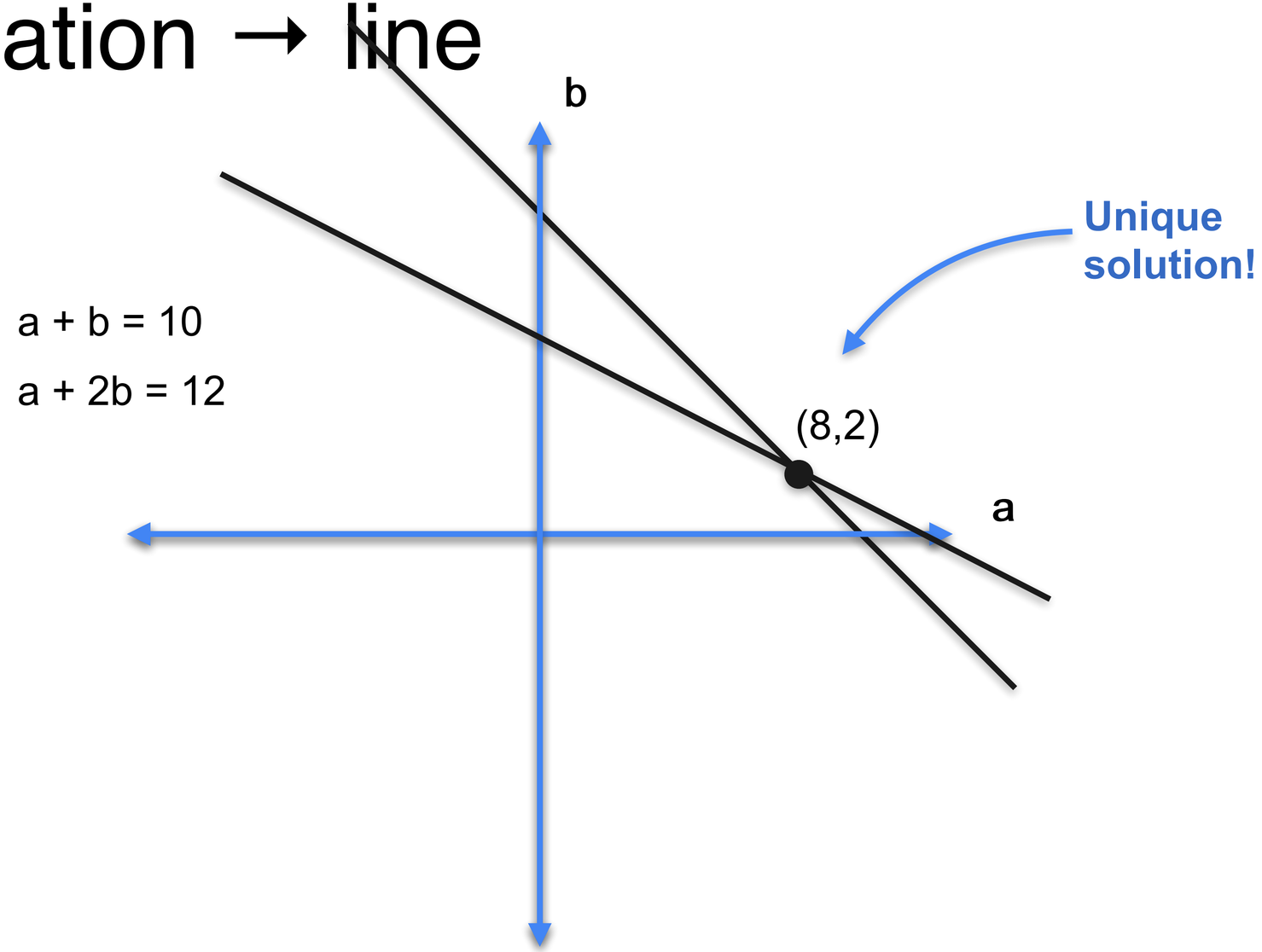


Linear equation → line





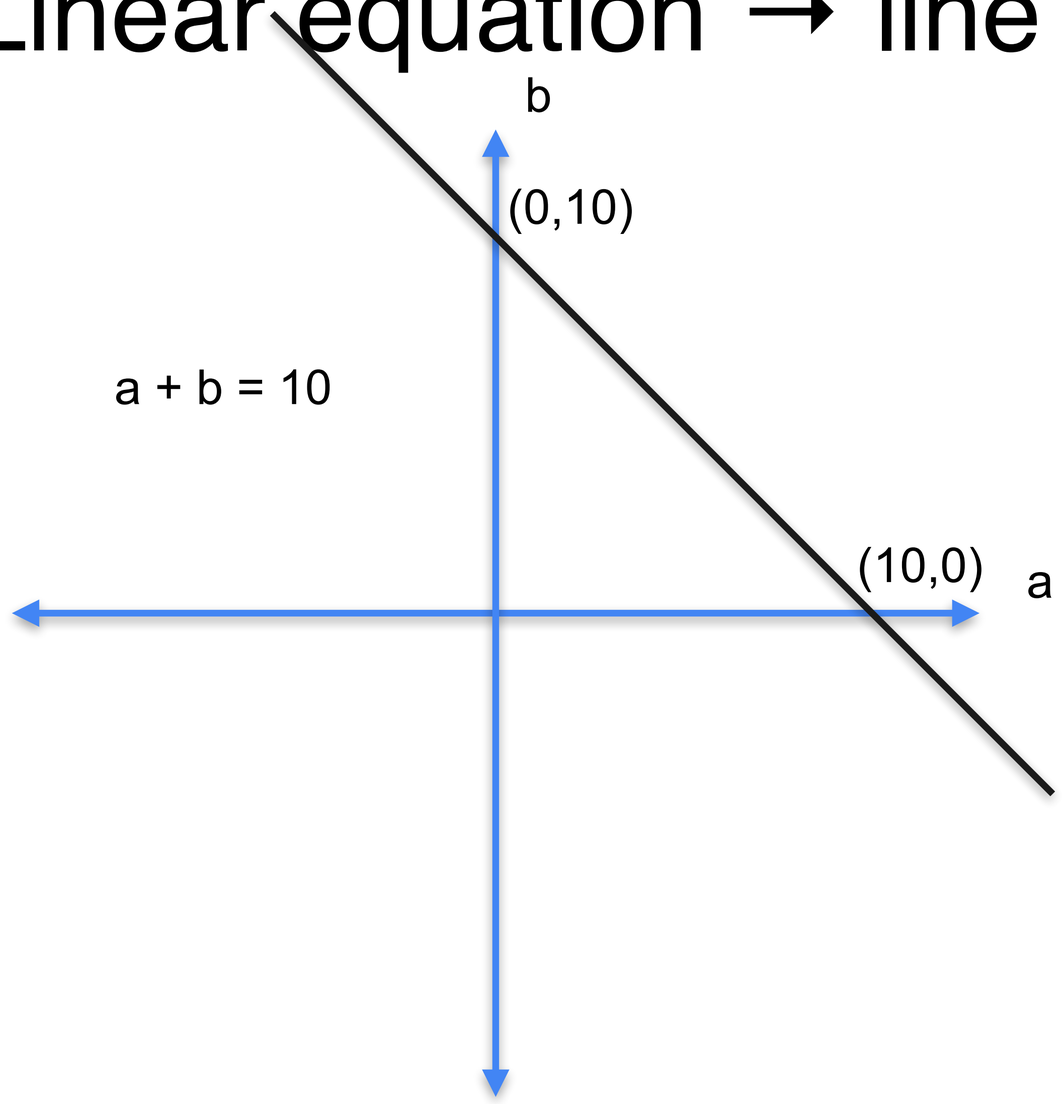
Linear equation → line

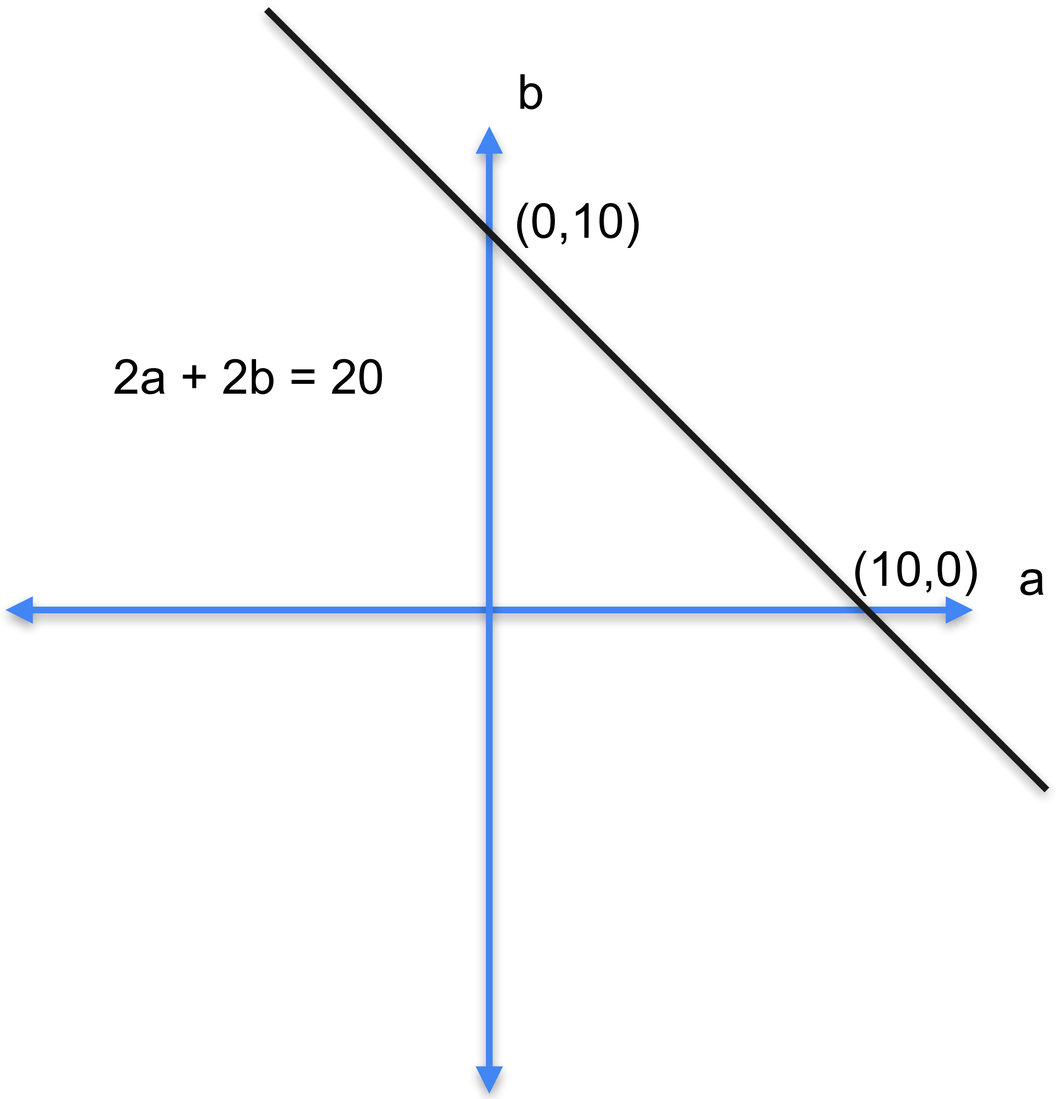


$a + b = 10$

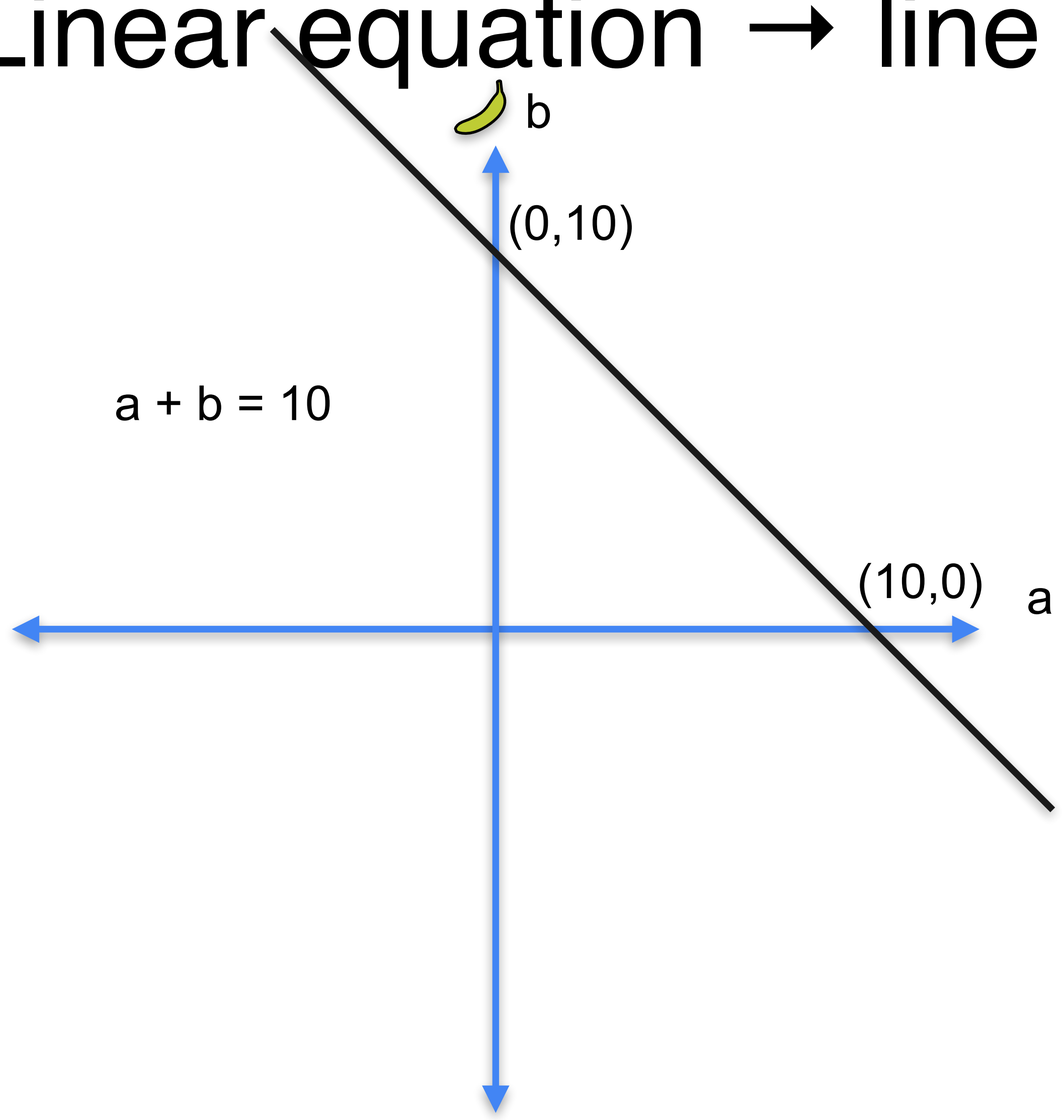
$a + 2b = 12$

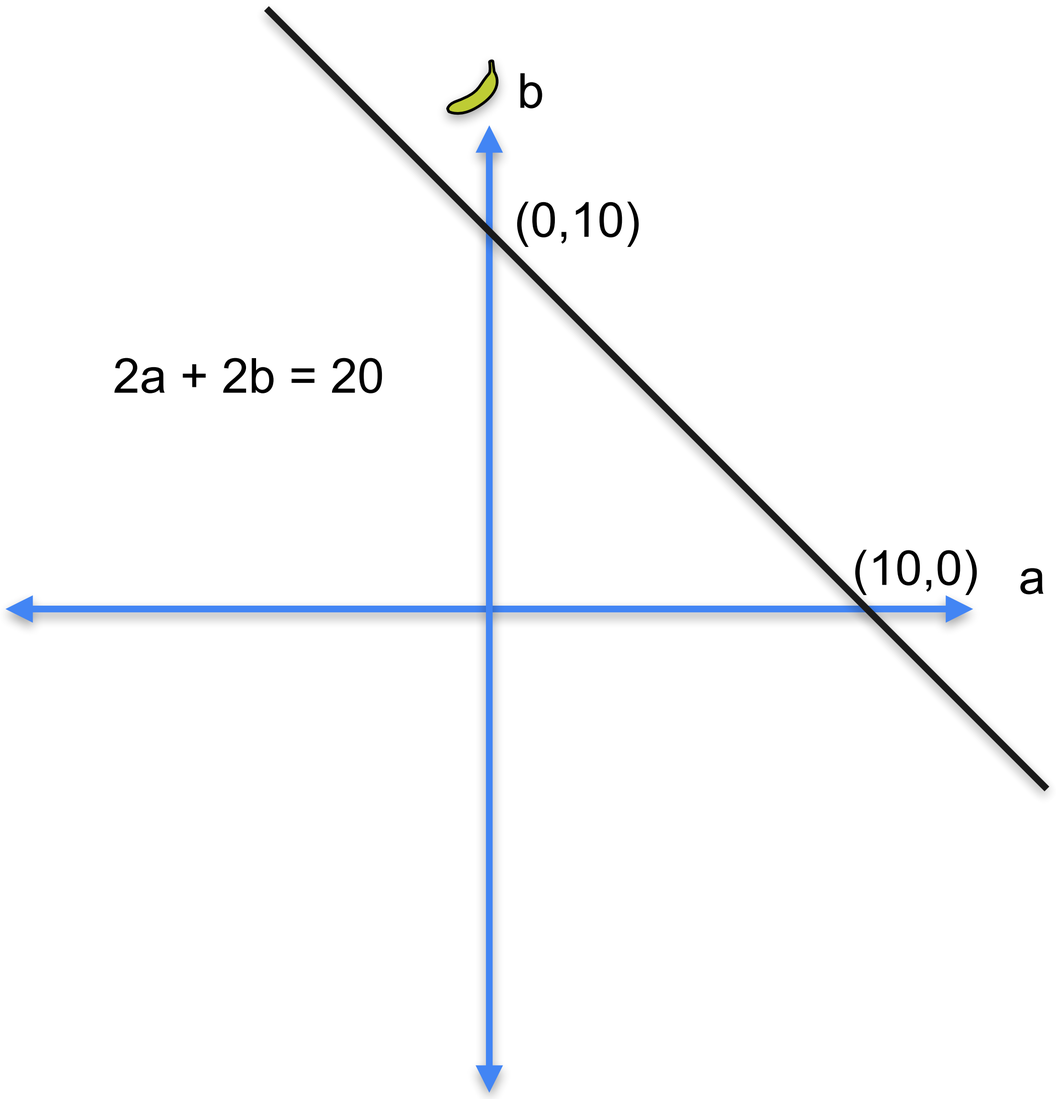
Linear equation → line





Linear equation → line

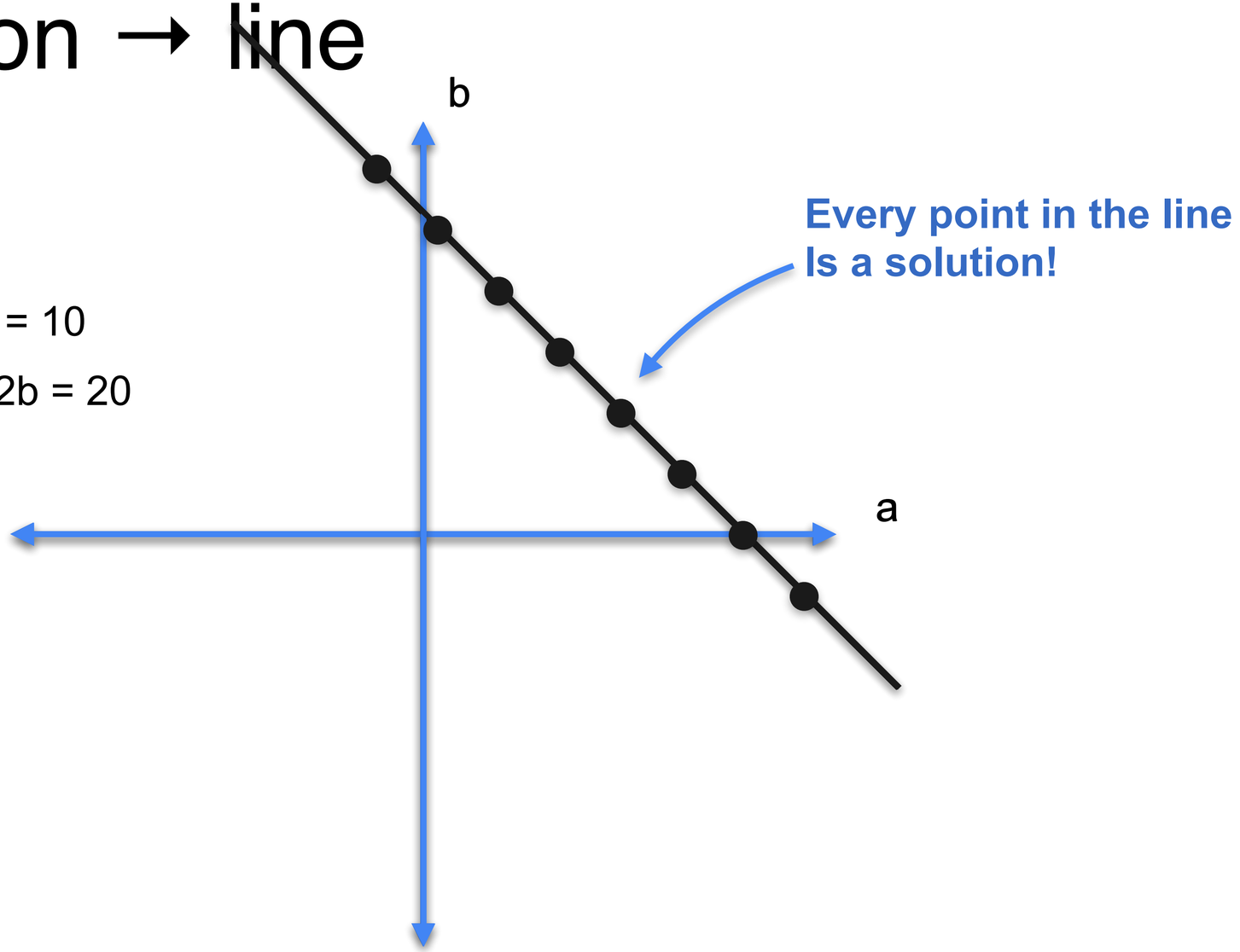




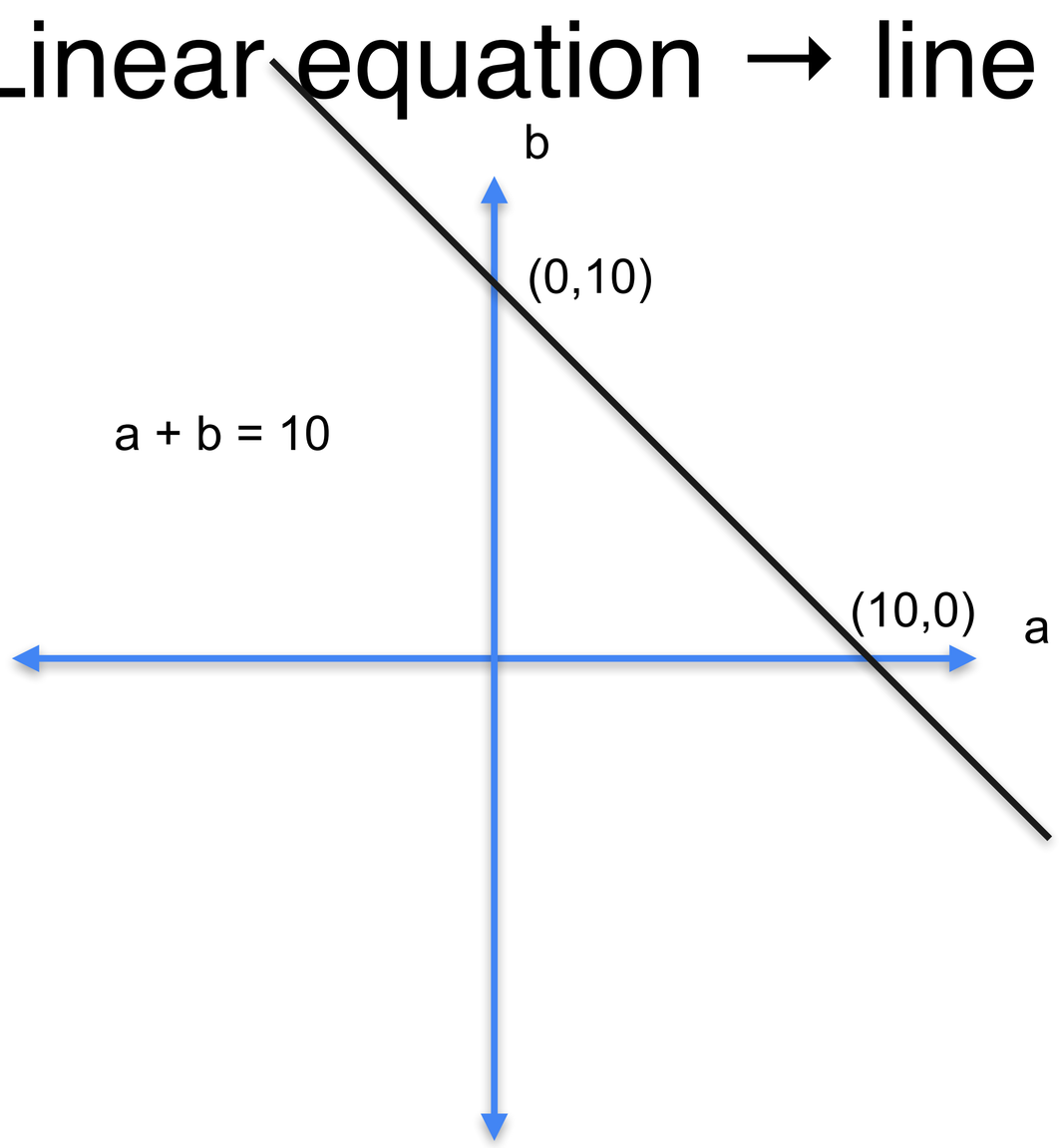
Linear equation → line

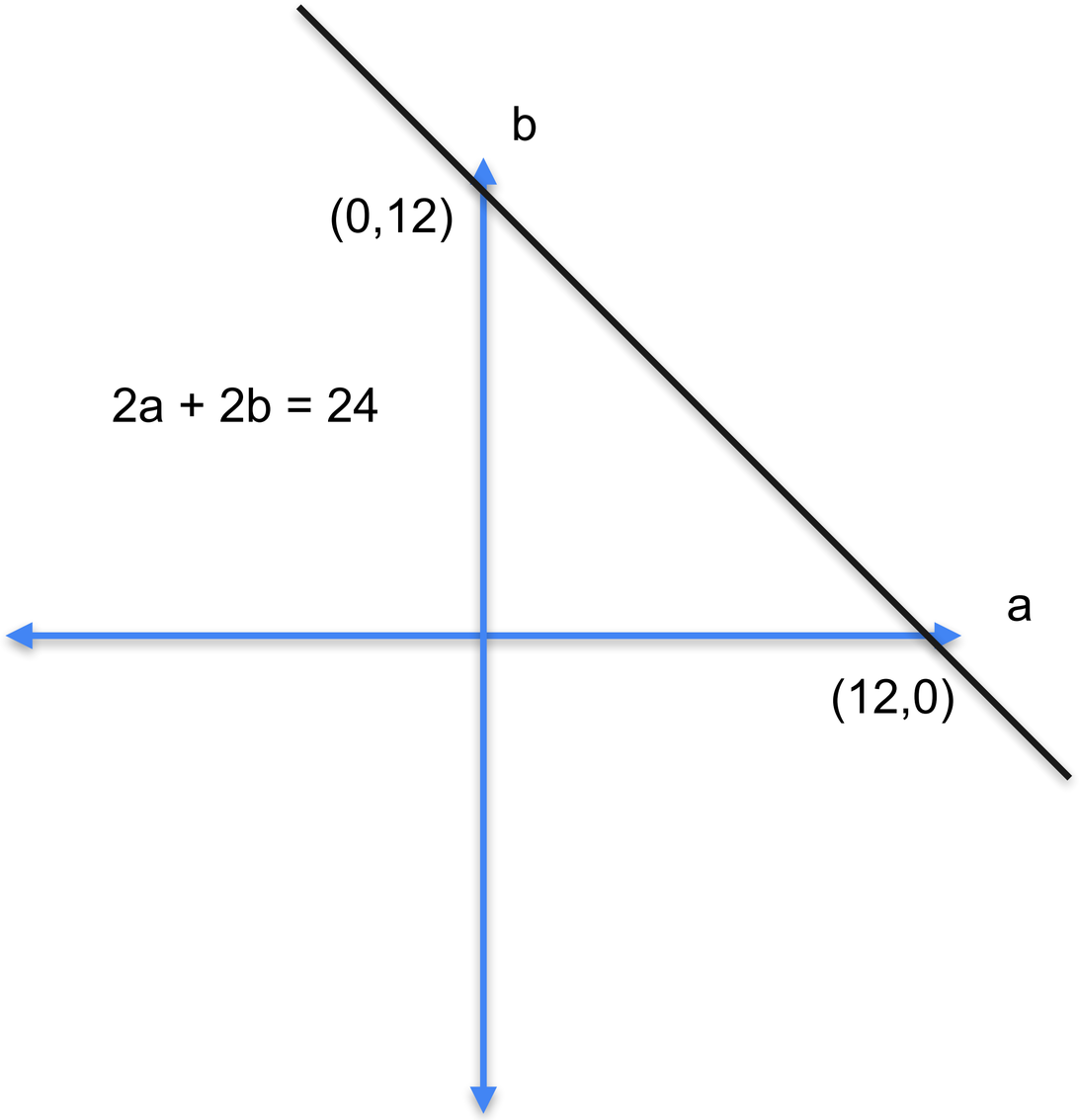
$a + b = 10$

$2a + 2b = 20$

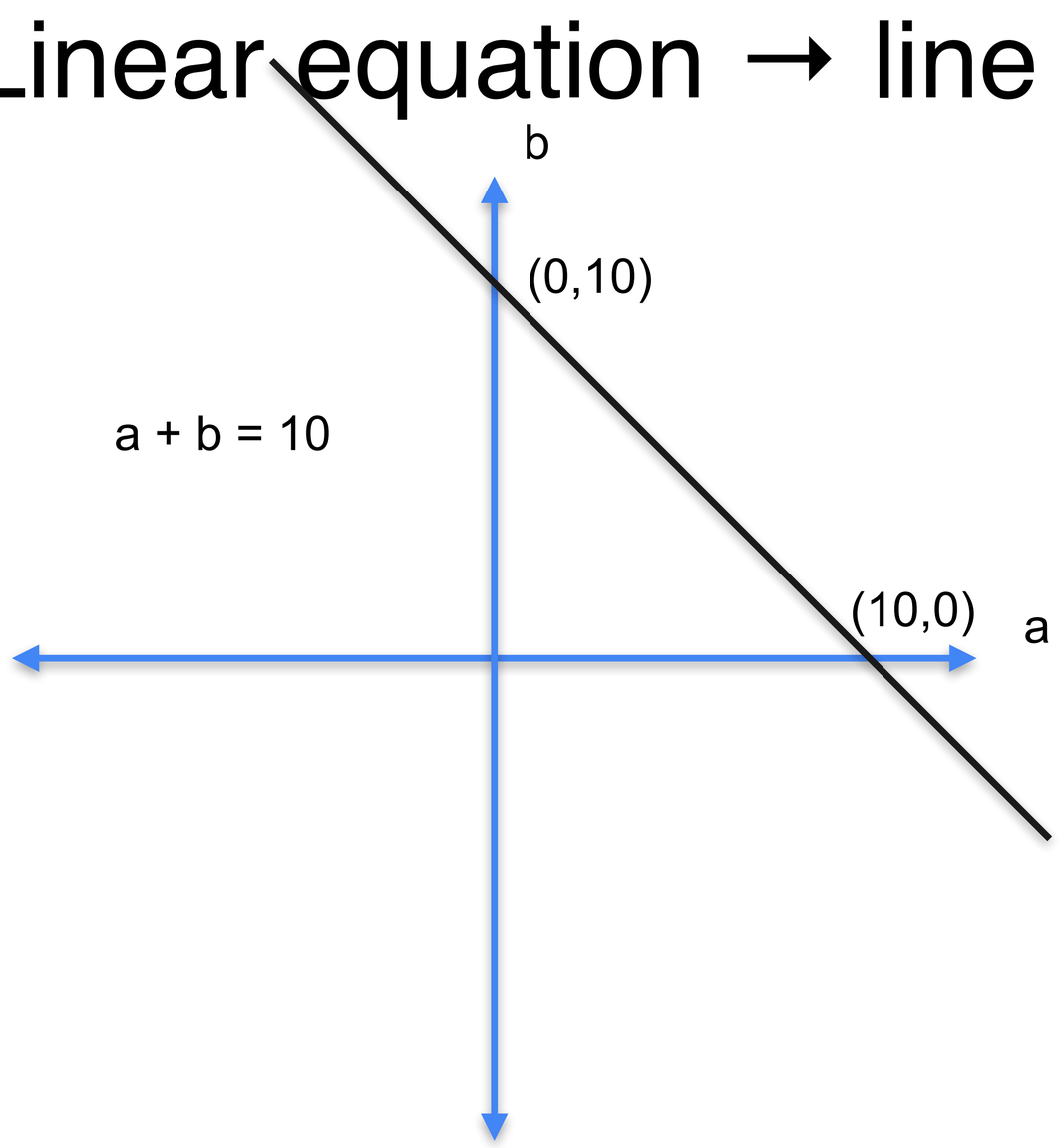


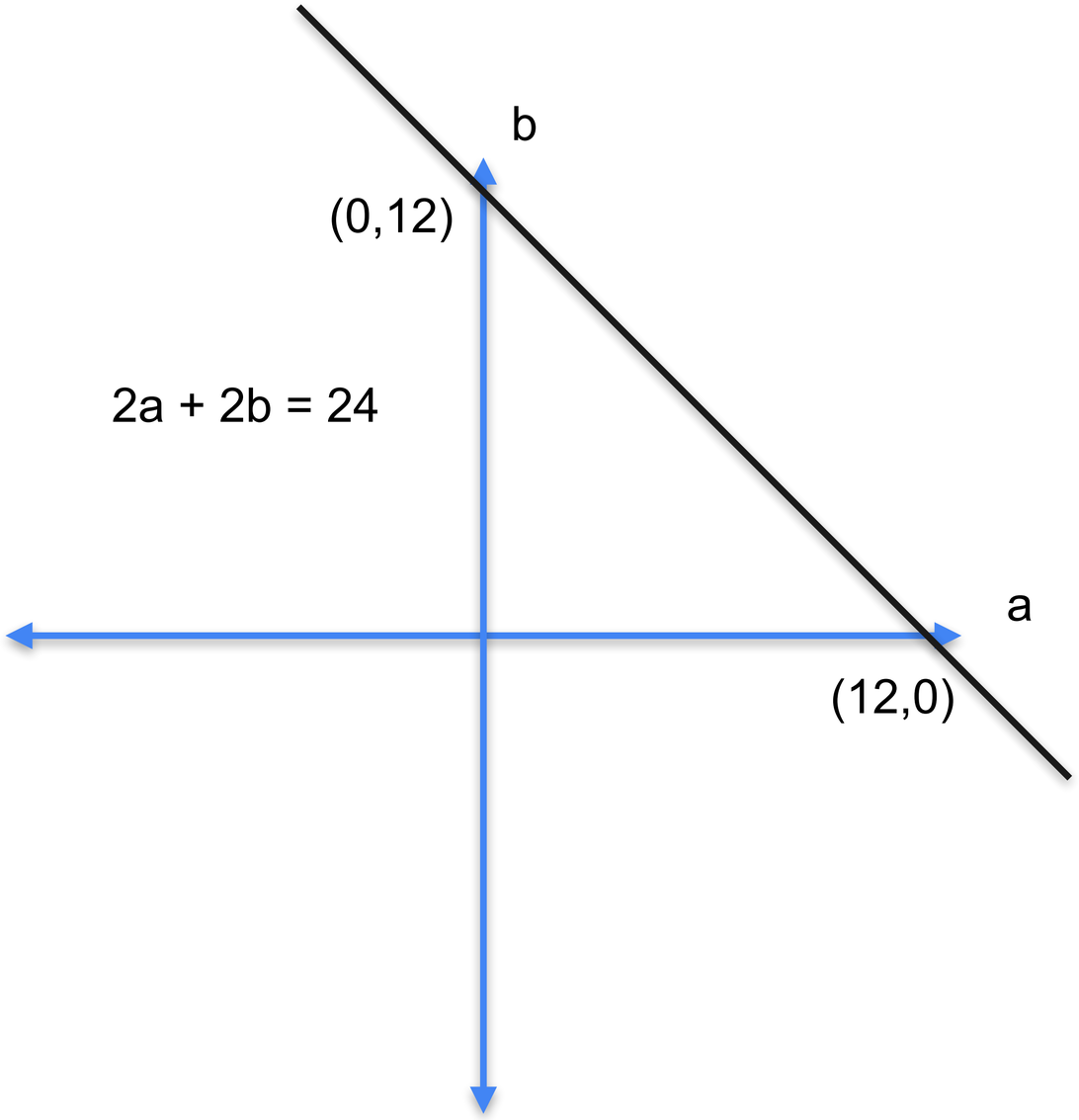
Linear equation → line





Linear equation → line

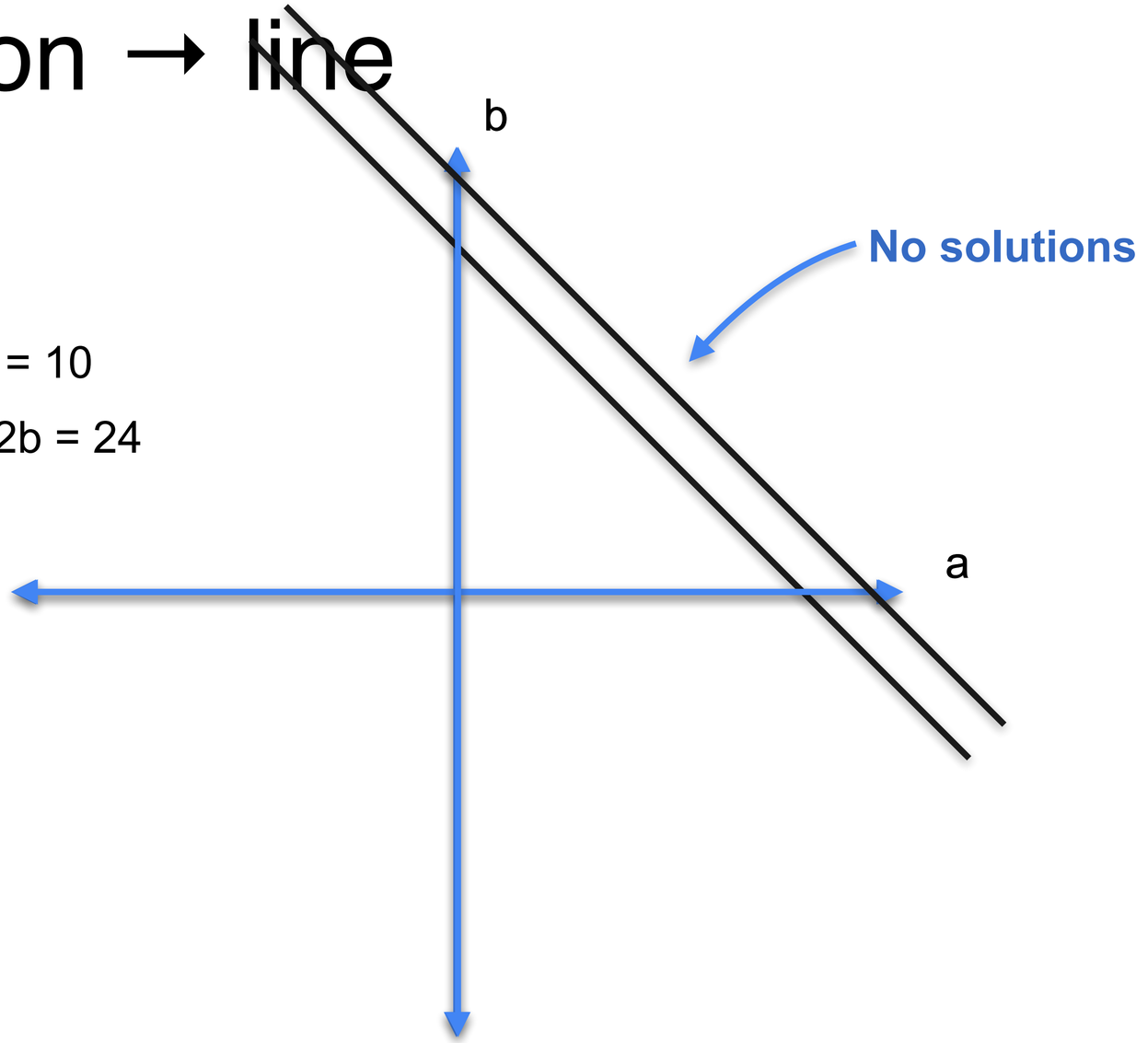




Linear equation → line

$a + b = 10$

$2a + 2b = 24$

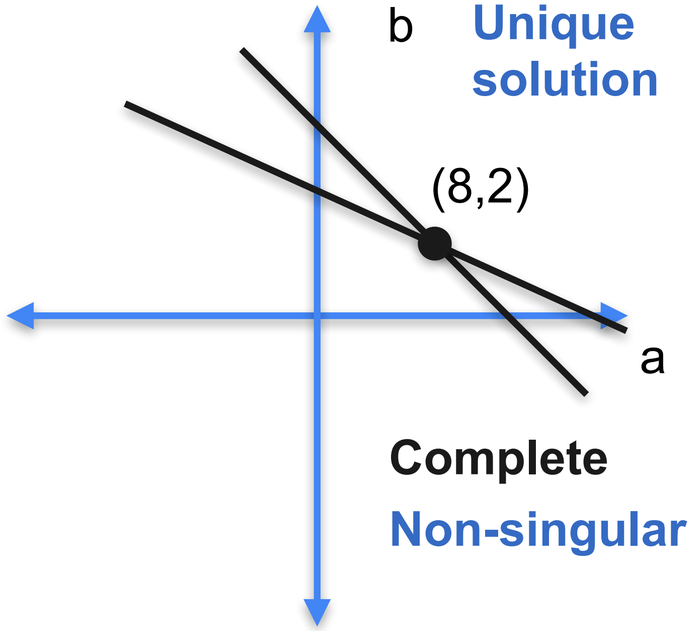


Systems of equations as lines

System 1

$a + b = 10$

$a + 2b = 12$



Unique solution

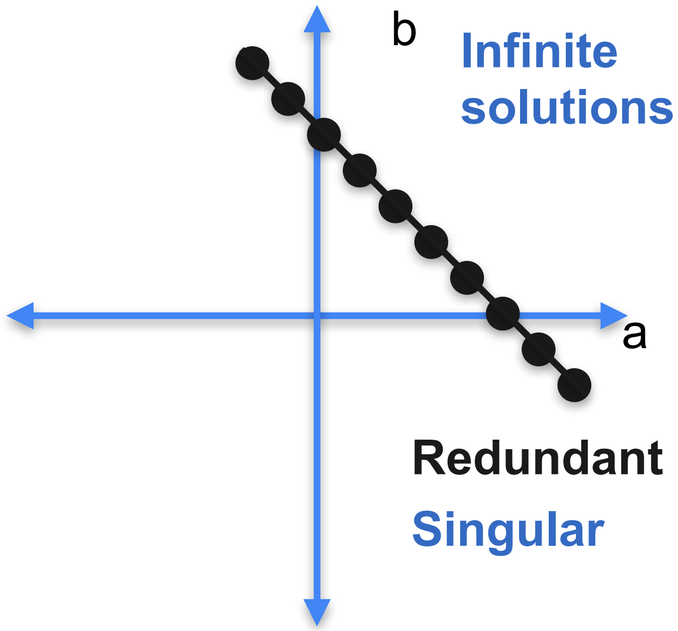
Complete

Non-singular

System 2

$a + b = 10$

$2a + 2b = 20$



Infinite solutions

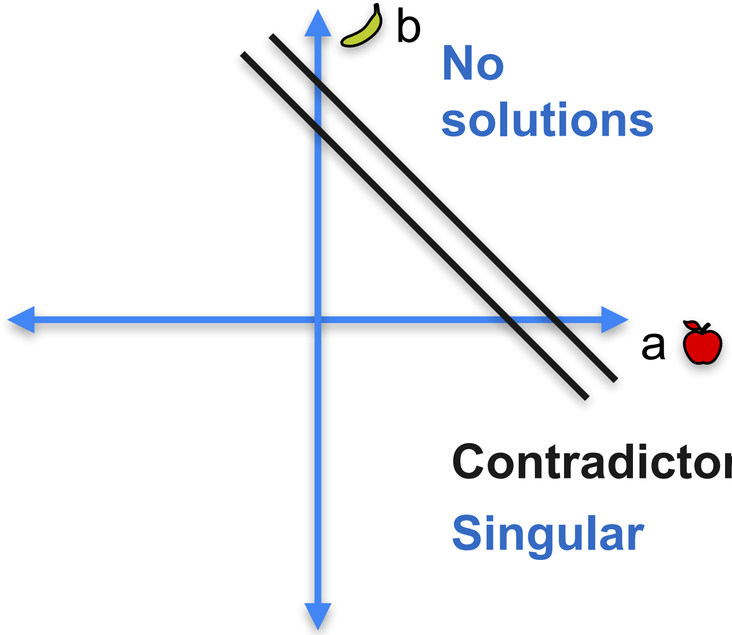
Redundant

Singular

System 3

$a + b = 10$

$2a + 2b = 24$



No solutions

Contradictory

Singular

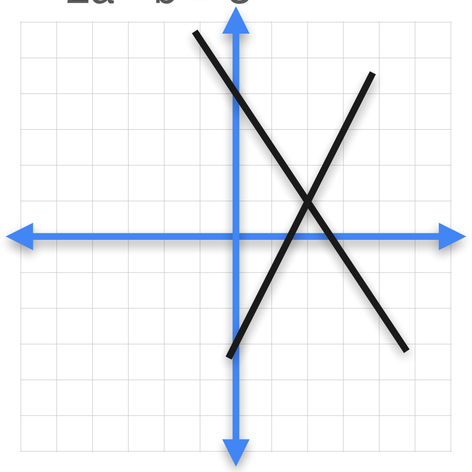
Quiz

Problem 1

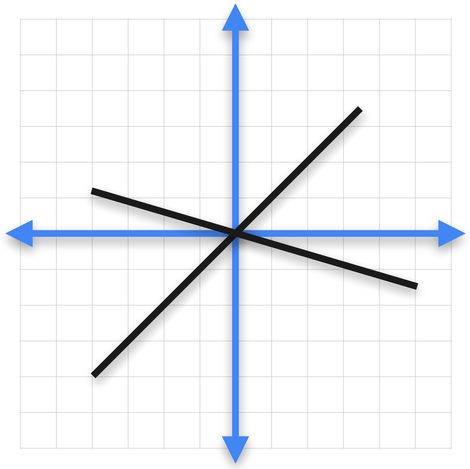
Which of the following plots corresponds to the system of equations:

- 3a + 2b = 8
- 2a - b = 3

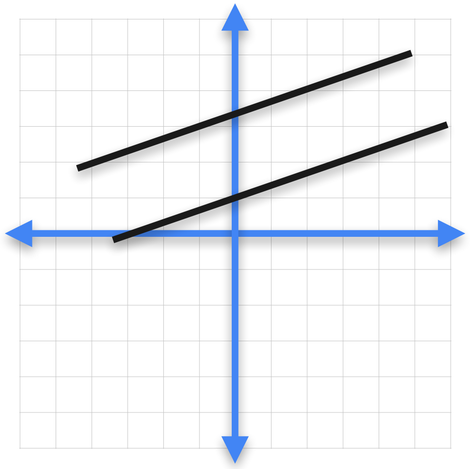
a)



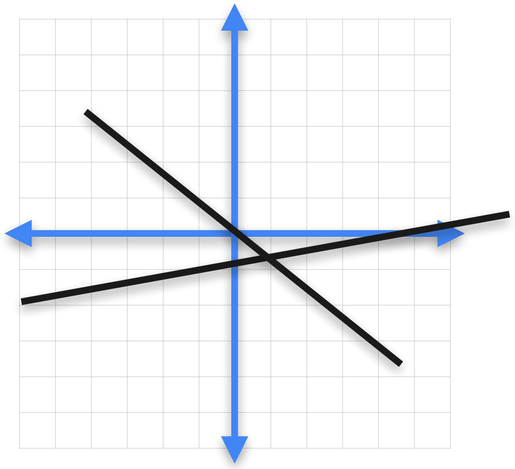
b)



c)



d)



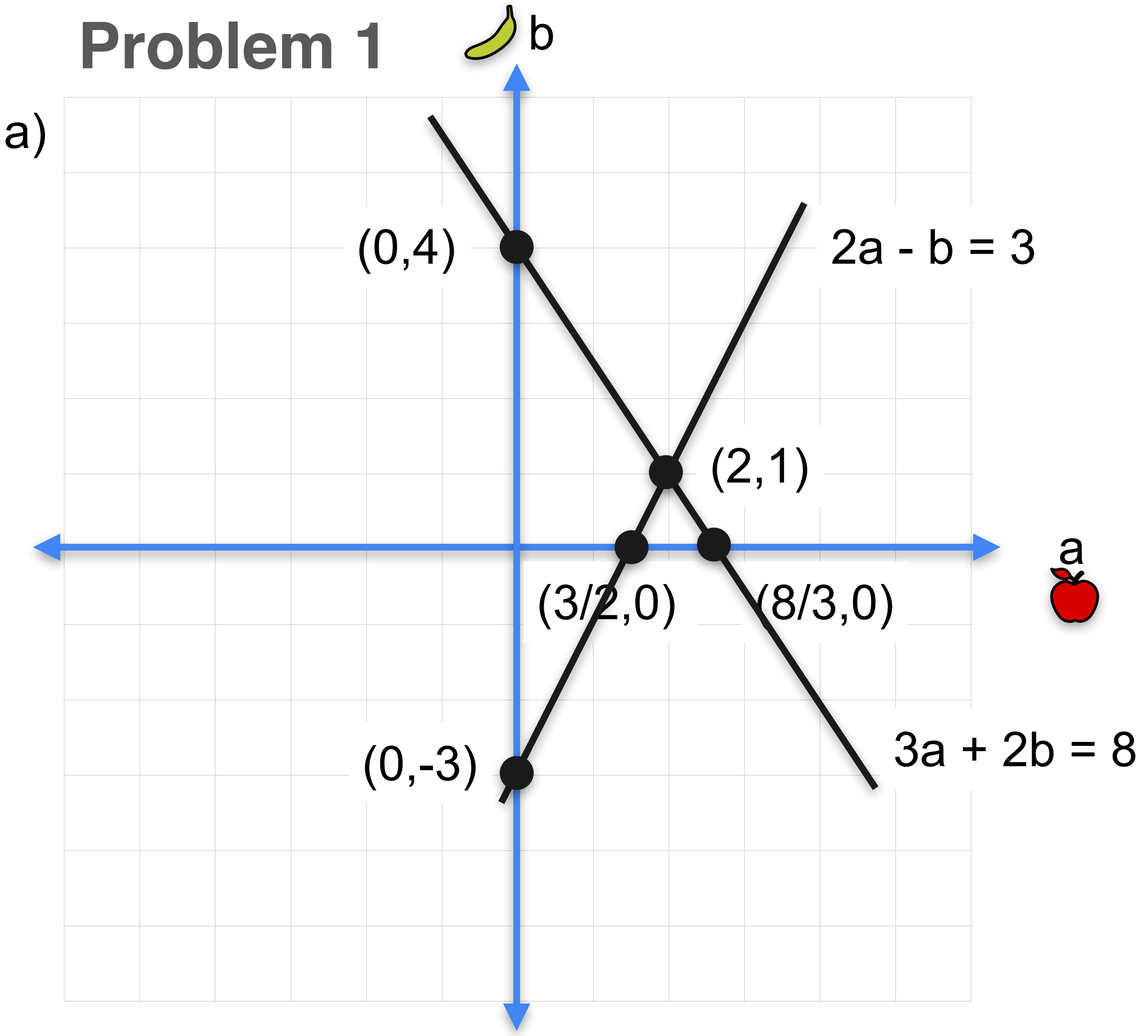
Problem 2

Is this system singular or non-singular?

Solution

Problem 1

a)



Problem 2

Since the lines cross at a unique point, the system is non-singular.

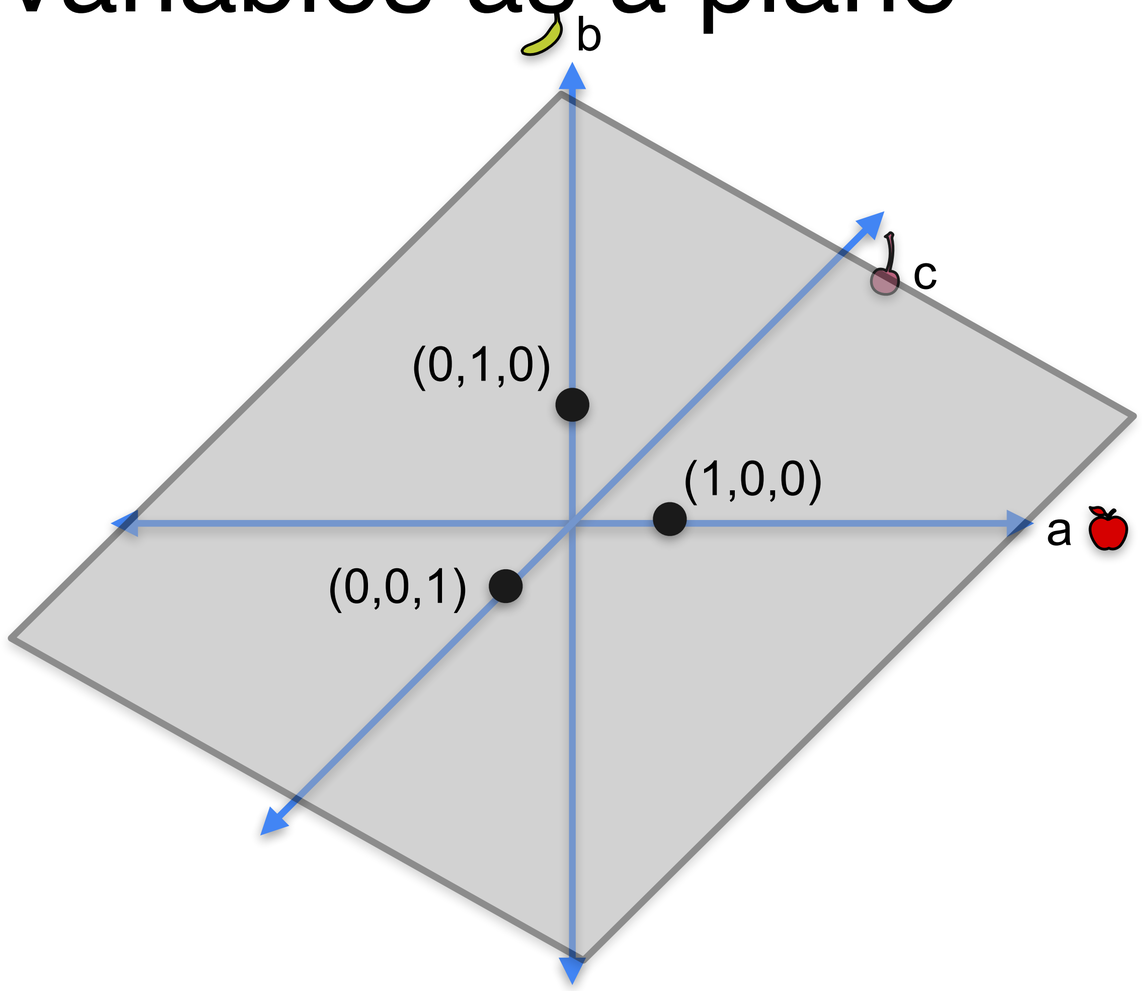
Linear equation in 3 variables as a plane

$a + b + c = 1$

$1 + 0 + 0 = 1$

$0 + 1 + 0 = 1$

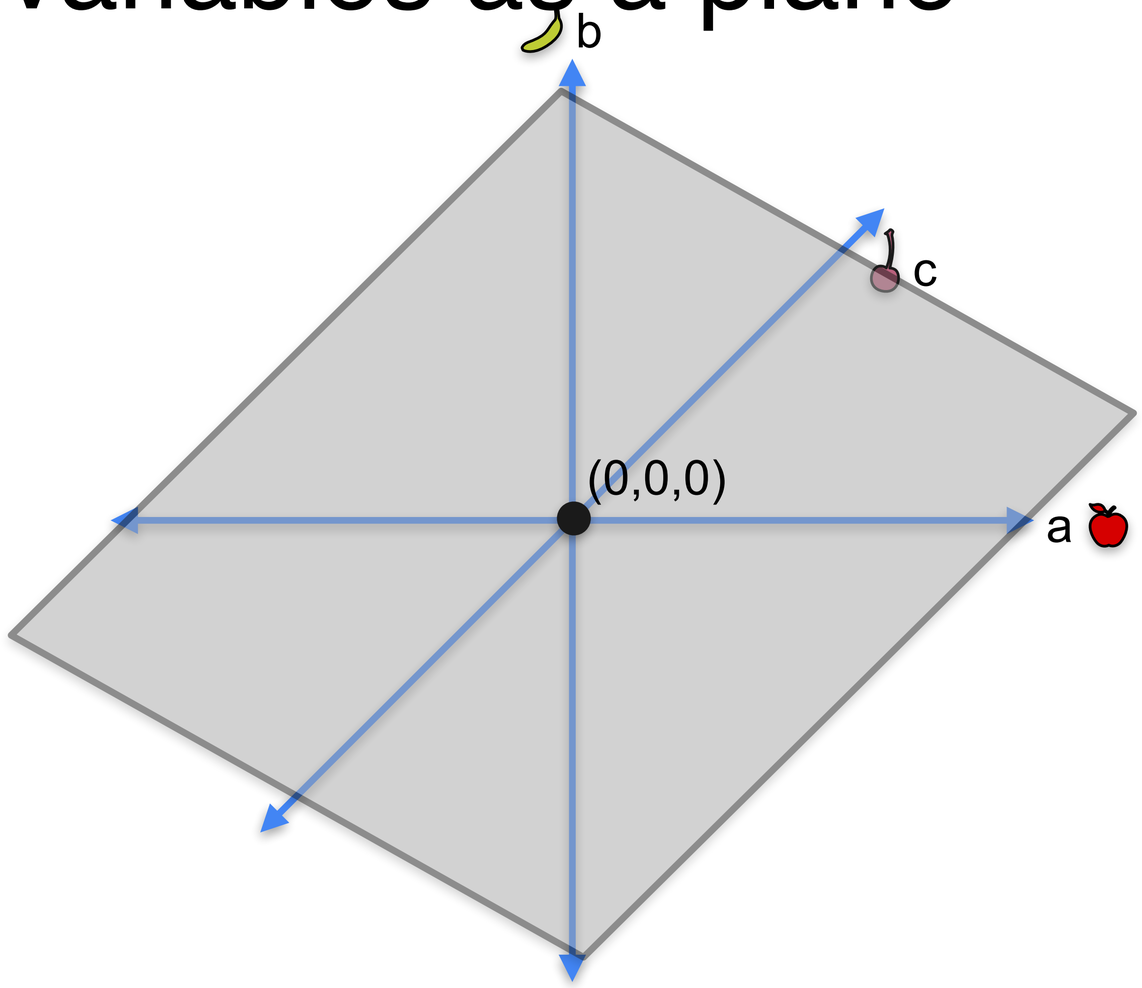
$0 + 0 + 1 = 1$



Linear equation in 3 variables as a plane

$3a - 5b + 2c = 0$

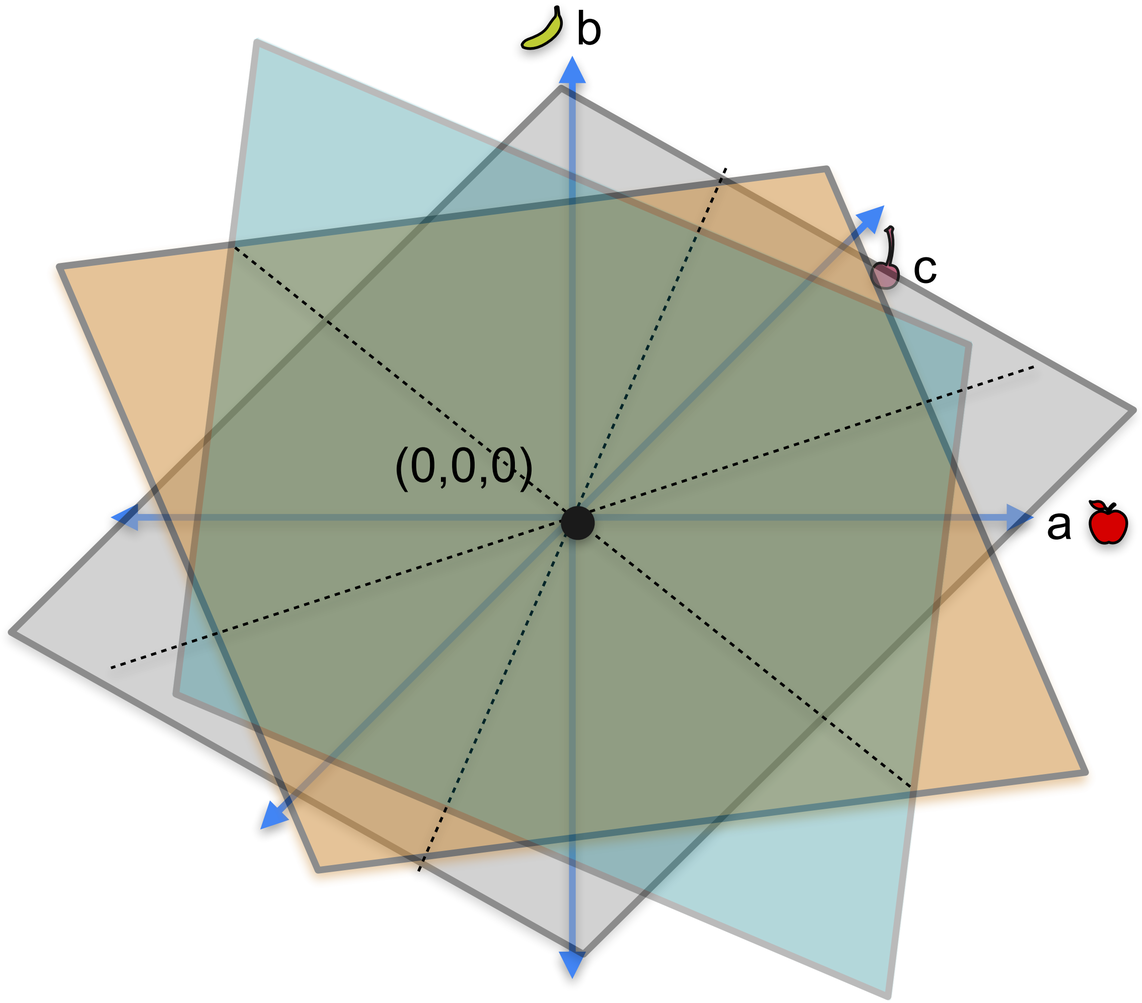
$3(0) + 5(0) + 2(0) = 0$



System 1

System 1

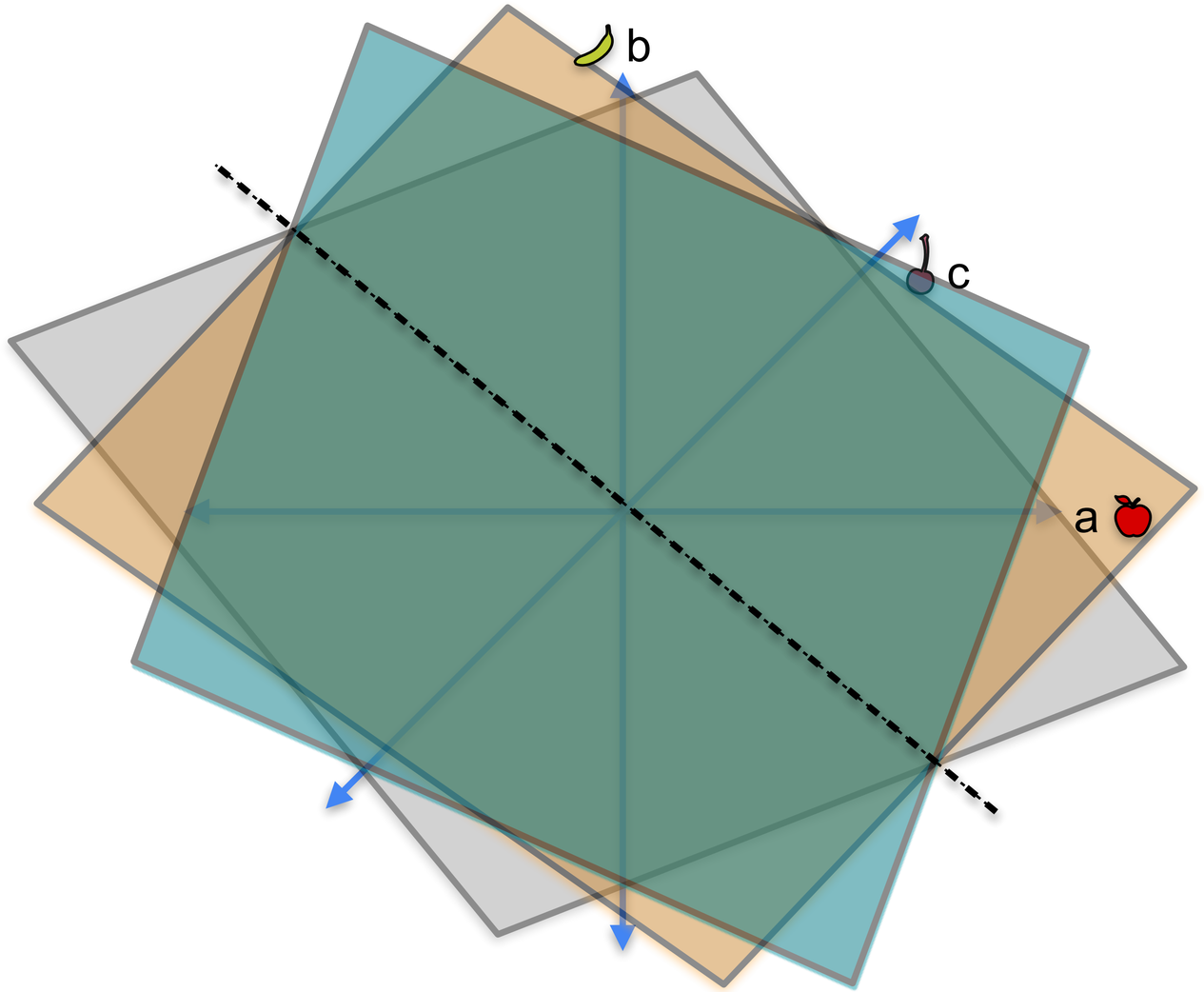
- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$



System 2

System 2

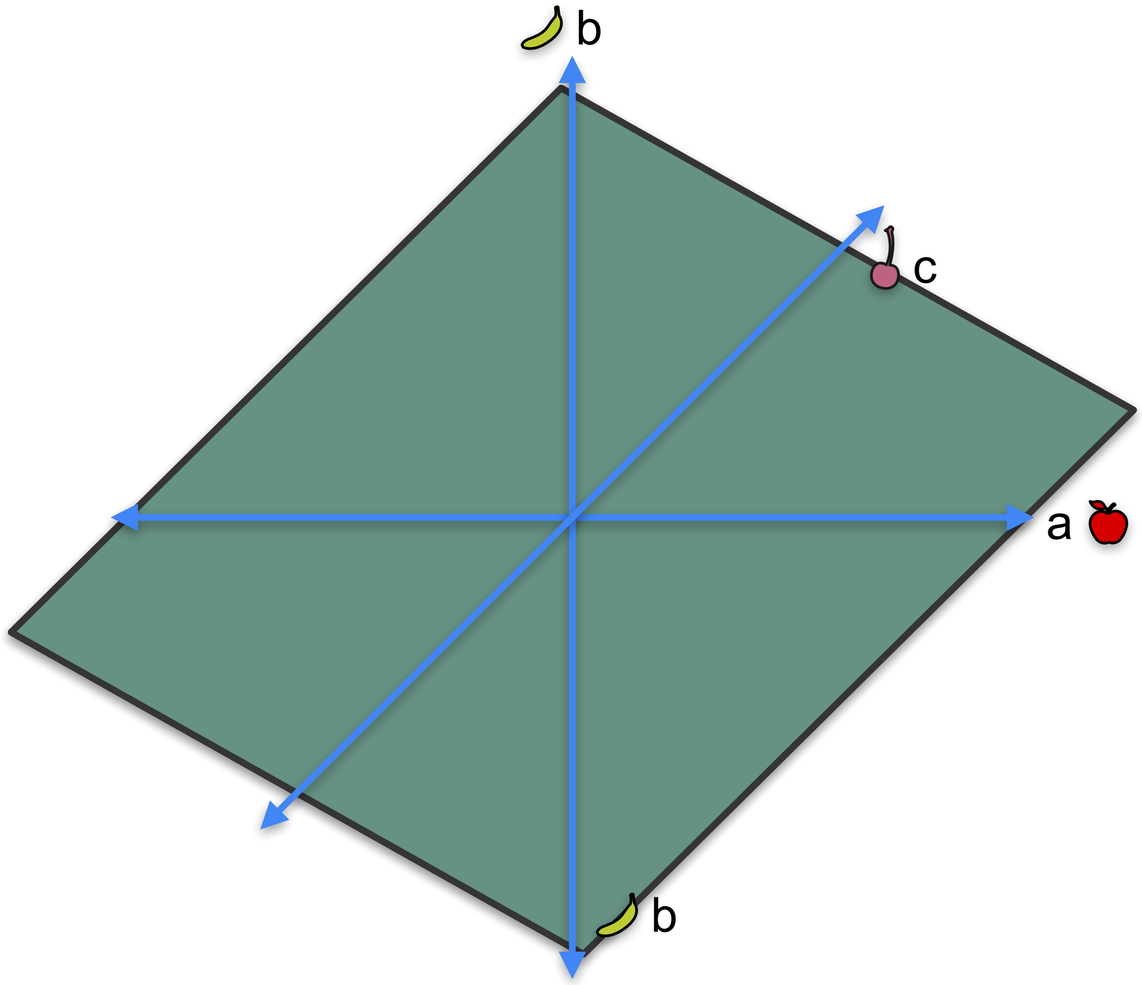
- a + b + c = **0**
- a + b + 2c = **0**
- a + b + 3c = **0**



System 3

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$



DeepLearning.AI

System of Linear Equations

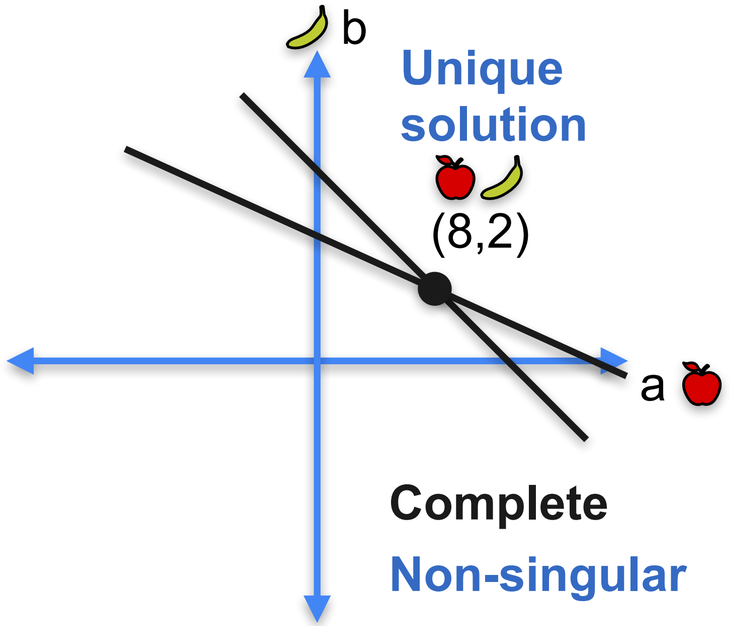
A geometric notion of singularity

Systems of equations as lines

System 1

a + b = 10

a + 2b = 12



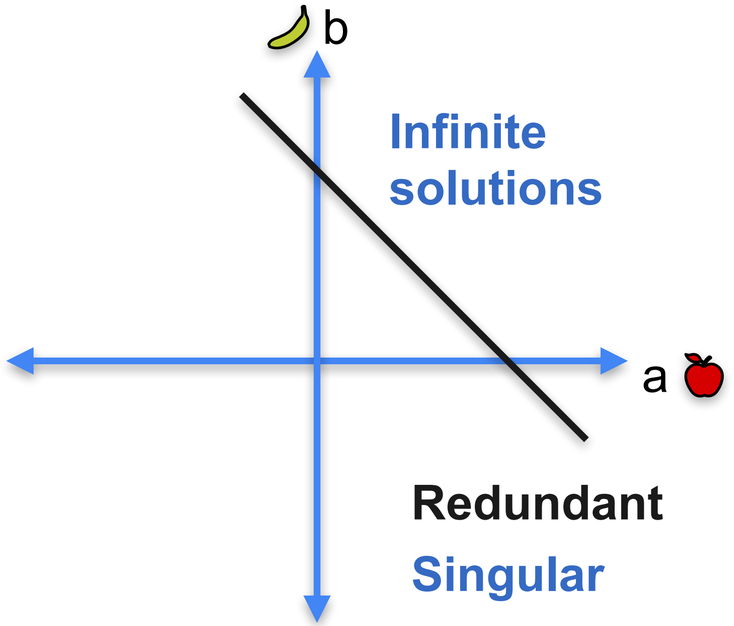
Complete

Non-singular

System 2

a + b = 10

2a + 2b = 20



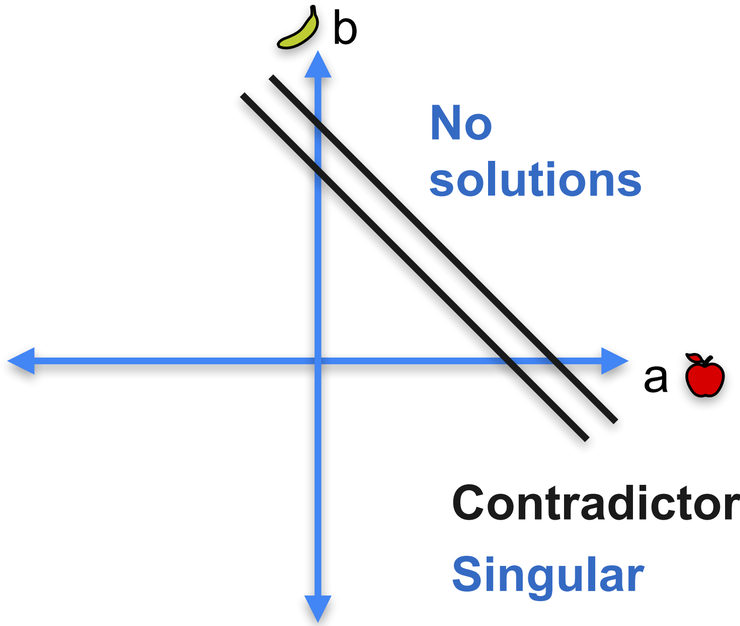
Redundant

Singular

System 3

a + b = 10

2a + 2b = 24



Contradictory

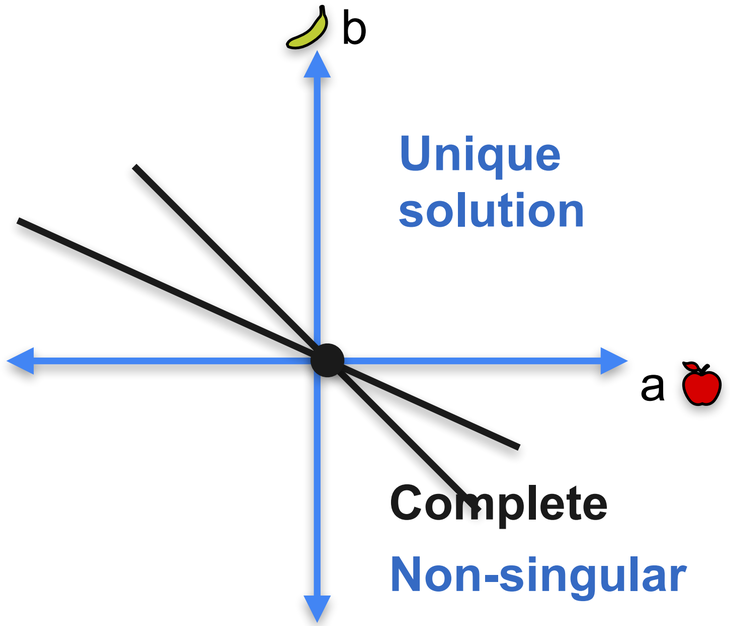
Singular

Systems of equations as lines

System 1

$a + b = 0$

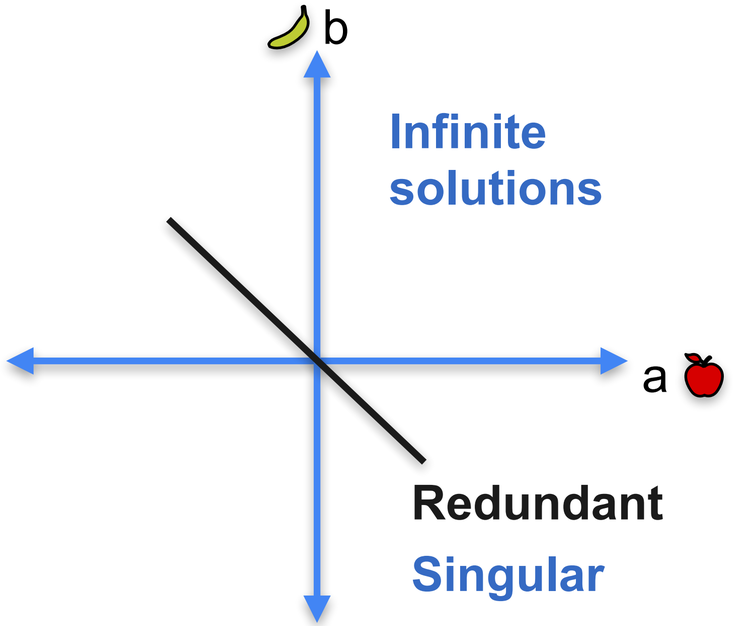
$a + 2b = 0$



System 2

$a + b = 0$

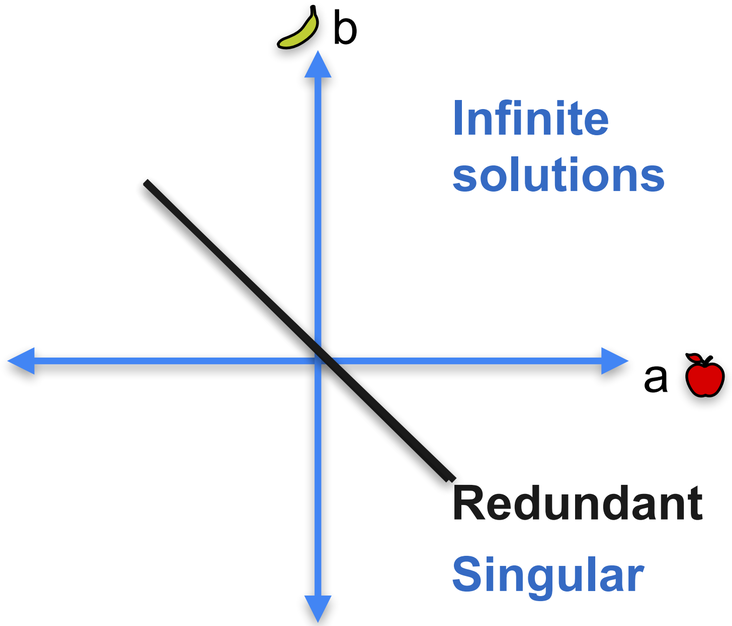
$2a + 2b = 0$



System 3

$a + b = 0$

$2a + 2b = 0$



DeepLearning.AI

System of Linear Equations

Singular vs non-singular matrices

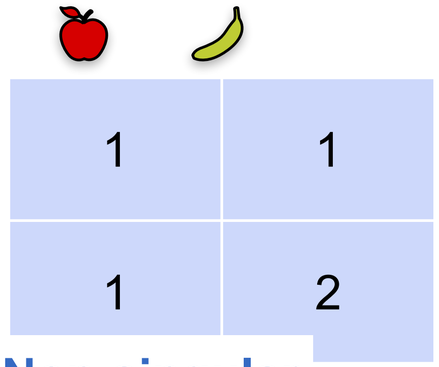
Systems of equations as matrices

System 1

a + b = 0

a + 2b = 0

Non-singular system



Non-singular matrix

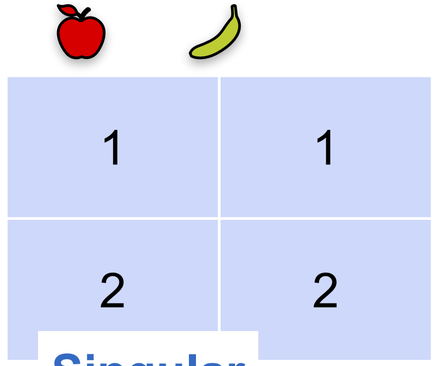
(Unique solution)

System 2

a + b = 0

2a + 2b = 0

Singular system



Singular matrix

(Infinitely many solutions)

Constants don’t matter for singularity

System 1

$a + b + c = 10$
$a + 2b + c = 15$
$a + b + 2c = 12$

Unique solution

Complete

Non-singular

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

Infinite solutions

Redundant

Singular

System 3

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

No solutions

Contradictory

Singular

System 4

$a + b + c = 10$
$2a + 2b + 2c = 15$
$3a + 3b + 3c = 20$

Infinite solutions

Redundant

Singular

Constants don’t matter for singularity

System 1

$a + b + c = 10$
$a + 2b + c = 15$
$a + b + 2c = 12$

$a + b + c = 0$
$a + 2b + c = 0$
$a + b + 2c = 0$

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 4

$a + b + c = 10$
$2a + 2b + 2c = 20$
$3a + 3b + 3c = 30$

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

Constants don’t matter for singularity

System 1

$a + b + c = 0$
$a + 2b + c = 0$
$a + b + 2c = 0$

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 4

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

Constants don’t matter for singularity

System 1

$a + b + c = 0$
$a + 2b + c = 0$
$a + b + 2c = 0$

1	1	1
1	2	1
1	1	2

Non-singular

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

1	1	1
1	1	2
1	1	3

Singular

System 3

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 4

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

1	1	1
2	2	2
3	3	3

Singular

DeepLearning.AI

System of Linear Equations

Linear dependence and independence

Linear dependence between rows

Non-singular

a + b = 0

a + 2b = 0

1	1
1	2

No equation is a multiple of the other one

No row is a multiple of the other one

Rows are *linearly independent*

Singular system

a + b = 0

2a + 2b = 0

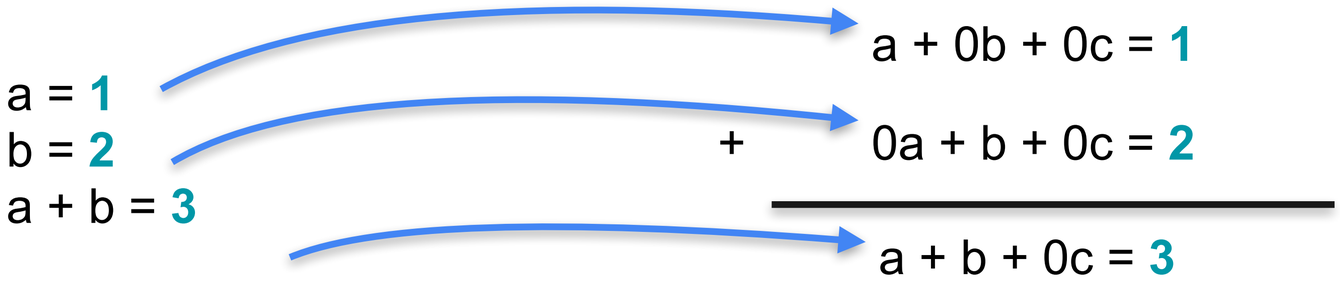
1	1
2	2

Second equation is a multiple of the first one

Second row is a multiple of the first row

Rows are *linearly dependent*

Linear dependence and independence



1	0	0
0	1	0
1	1	0

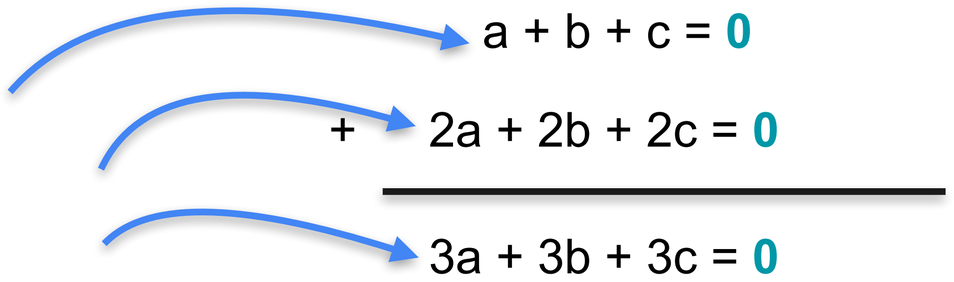
Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

Linear dependence and independence

$$
\begin{aligned}
a + b + c &= 0 \\
2a + 2b + 2c &= 0 \\
3a + 3b + 3c &= 0
\end{aligned}
$$



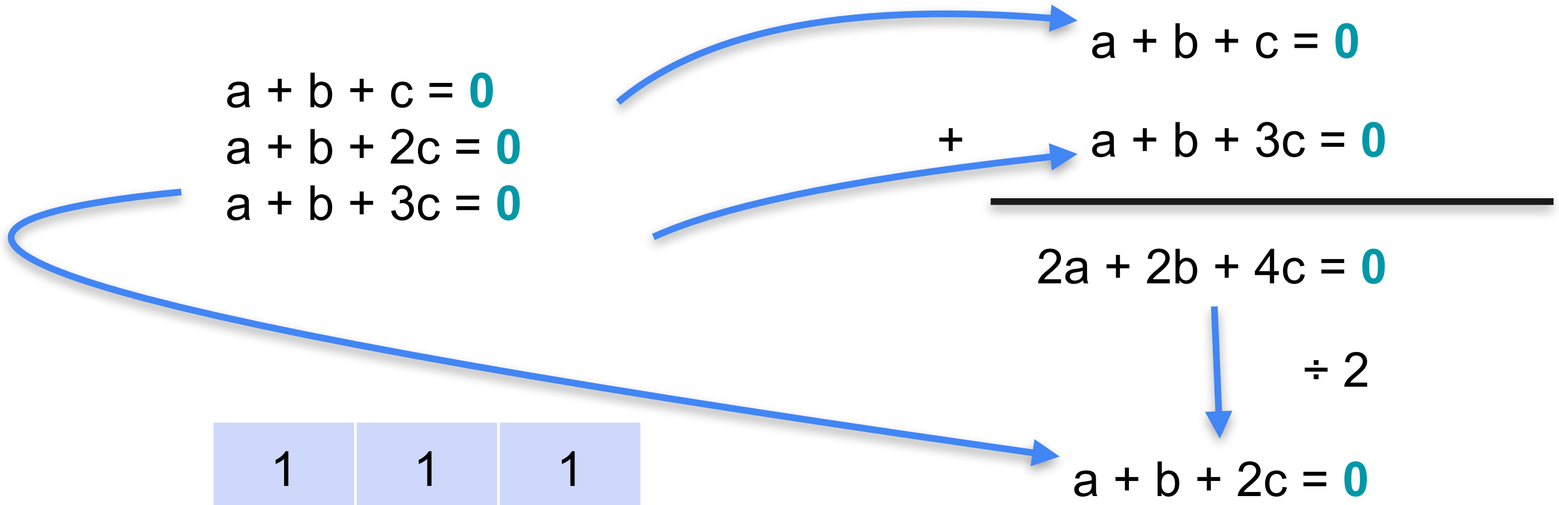
1	1	1
2	2	2
3	3	3

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

Linear dependence and independence



1	1	1
1	1	2
1	1	3

Average of Row 1 and Row 3 is Row 2
Row 2 **depends** on rows 1 and 3

Rows are **linearly dependent**

Linear dependence and independence

$a + b + c = \mathbf{0}$
$a + 2b + c = \mathbf{0}$
$a + b + 2c = \mathbf{0}$

No relations between equations

1	1	1
1	2	1
1	1	2

No relations between rows

Rows are **linearly independent**

Quiz: Linear dependence and independence

Problem: Determine if the following matrices have linearly dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

Solution: Linear dependence and independence

Problem: Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

3Row1 + 2Row2 = Row3

Dependent (singular)

1	1	1
1	1	2
0	0	-1

Row1 - Row2 = Row3

Dependent (singular)

1	1	1
0	2	2
0	0	3

No relations

Independent (Non-singular)

1	2	5
0	3	-2
2	4	10

2Row1 = Row3

Dependent (singular)

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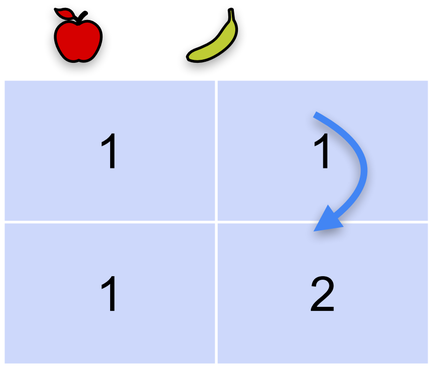
System of Linear Equations

The determinant

Linear dependence between rows

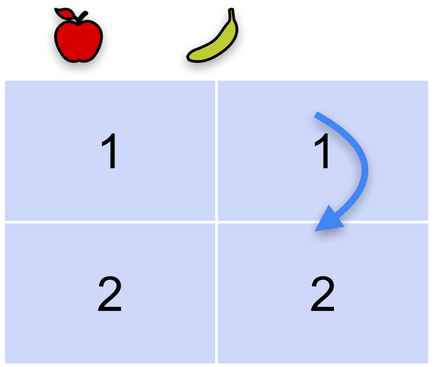
Non-singular matrix



1	1	x ?	=	1	2

Rows linearly independent

Singular matrix



1	1	x 2	=	2	2

Rows linearly dependent

Determinant

a	b
c	d

Determinant $= ad - bc$

a d - b c

Matrix is singular if

a	b	* k =	c	d

Determinant

$ad - bc = 0$

$ak = c$

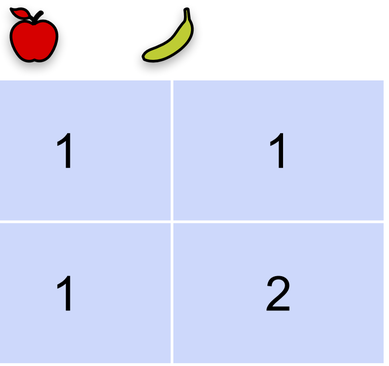
$bk = d$

$\frac{c}{a} = \frac{d}{b} = k$

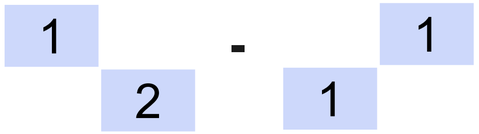
$ad = bc$

Determinant

Non-singular matrix

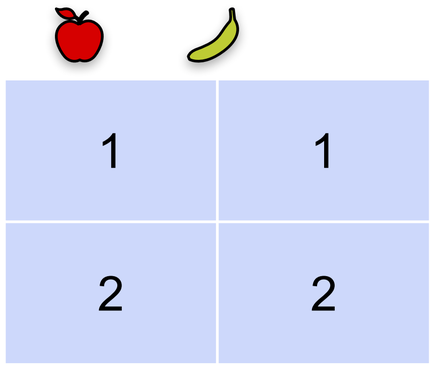


Determinant

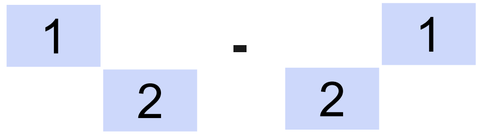


$1 \cdot 2 - 1 \cdot 1 = 1$

Singular matrix

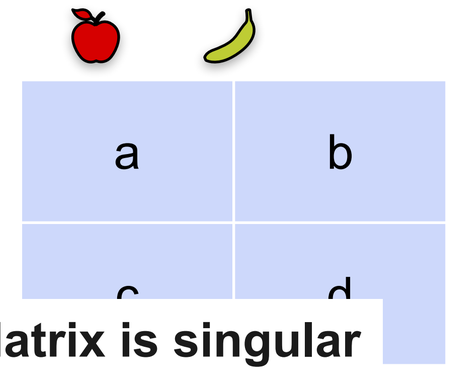


Determinant



$1 \cdot 2 - 2 \cdot 1 = 0$

Determinant and singularity



Matrix is singular

$ad - bc$

Determinant is zero

Quiz: Determinant

Problem 1: Find the determinant of the following matrices

Matrix 1

5	1
-1	3

Matrix 2

2	-1
-6	3

e these matrices singular or non-singular?

Solutions: Determinant

Matrix 1: det = $5 \cdot 3 - 1 \cdot (-1) = 15 + 1 = 16$

5	1
-1	3

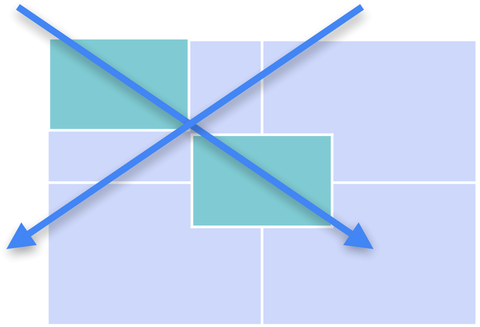
Non-singular

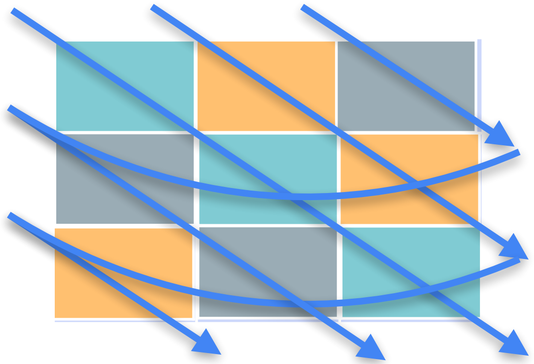
Matrix 2: det = $2 \cdot 3 - (-1) \cdot (-6) = 6 - 6 = 0$

2	-1
-6	3

Singular

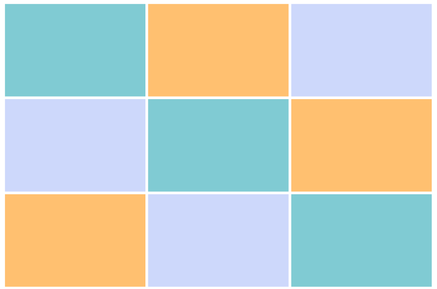
Diagonals in a 3x3 matrix



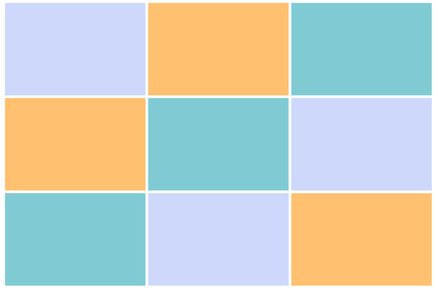


Determinant

Add



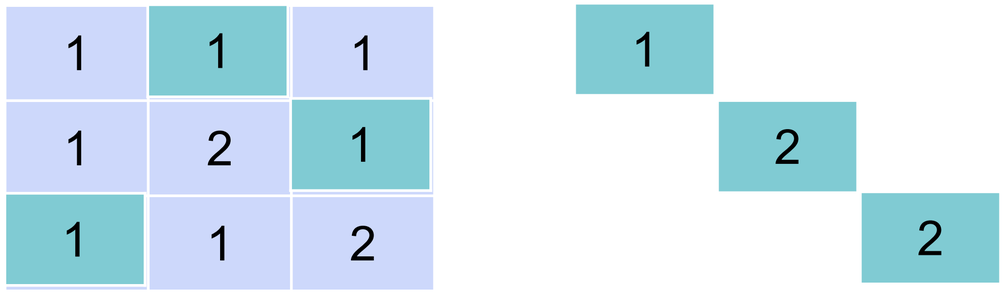
Subtract



The determinant

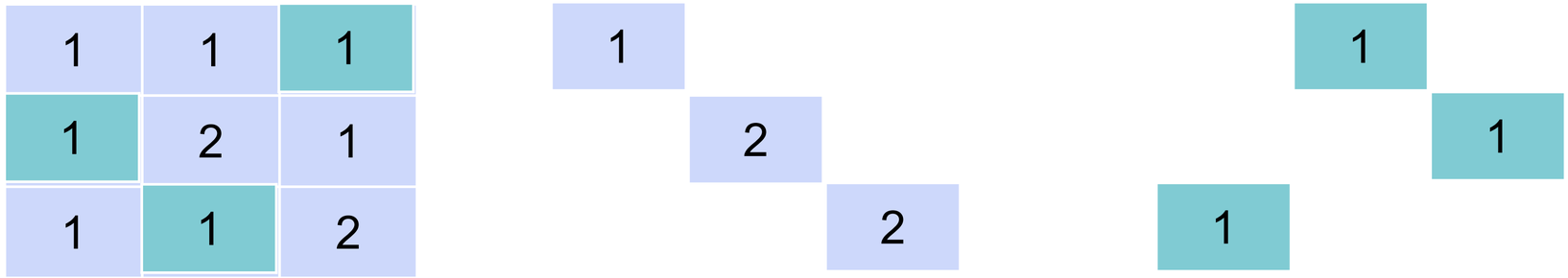
1	1	1
1	2	1
1	1	2

The determinant



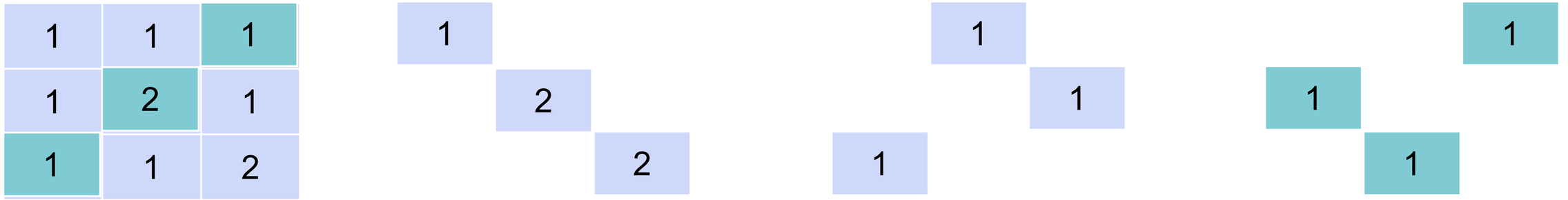
$$+ \quad 1 \cdot 2 \cdot 2$$

The determinant

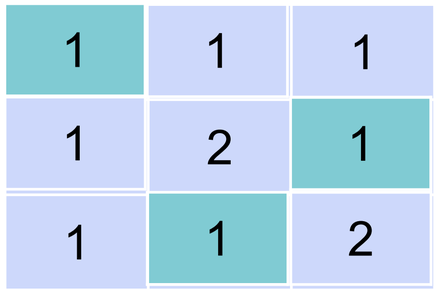


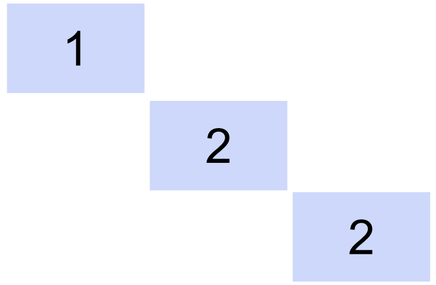
$$+ \; 1 \cdot 2 \cdot 2 \qquad + \; 1 \cdot 1 \cdot 1$$

The determinant

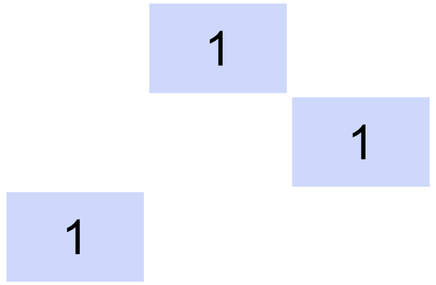


The determinant

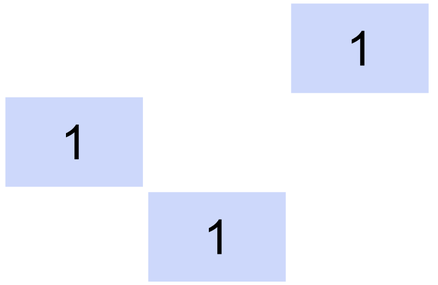




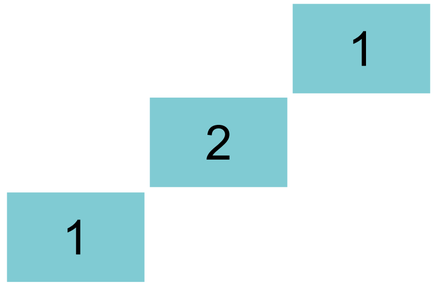
$+ \quad 1 \cdot 2 \cdot 2$



$+ \quad 1 \cdot 1 \cdot 1$

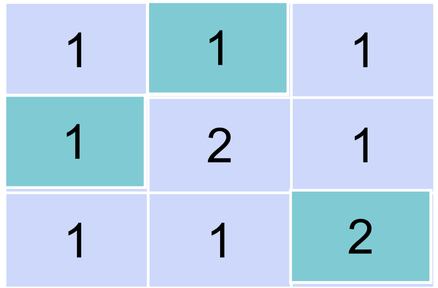


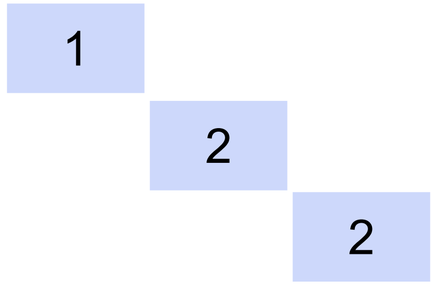
$+ \quad 1 \cdot 1 \cdot 1$



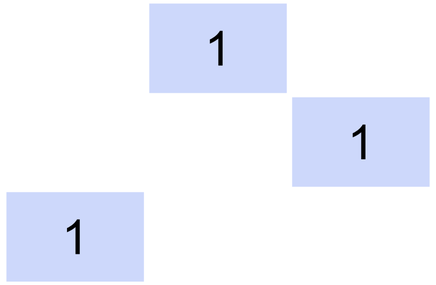
$- \quad 1 \cdot 2 \cdot 1$

The determinant

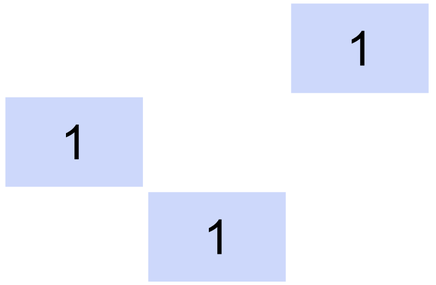




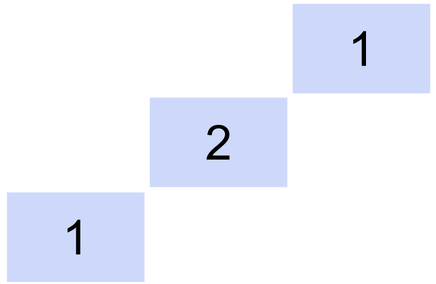
$$+ \; 1 \cdot 2 \cdot 2$$



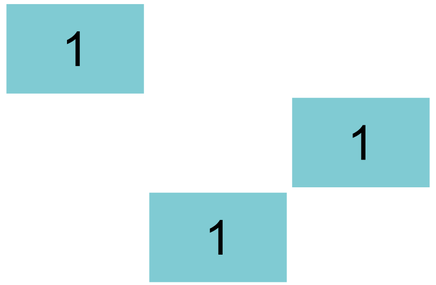
$$+ \; 1 \cdot 1 \cdot 1$$



$$+ \; 1 \cdot 1 \cdot 1$$



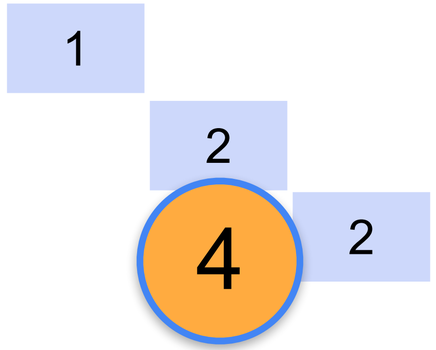
$$- \; 1 \cdot 2 \cdot 1$$



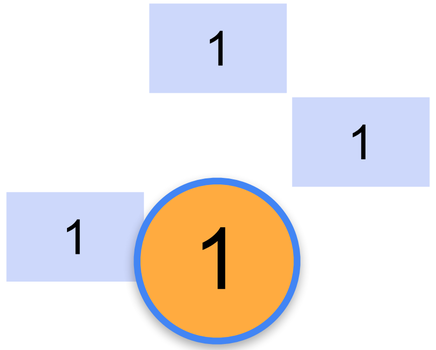
$$- \; 1 \cdot 1 \cdot 1$$

The determinant

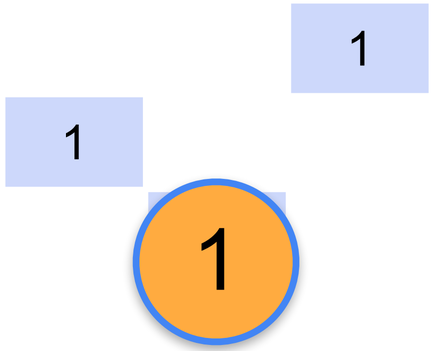
1	1	1
1	2	1
1	1	2



$+ \quad 1 \cdot 2 \cdot 2$



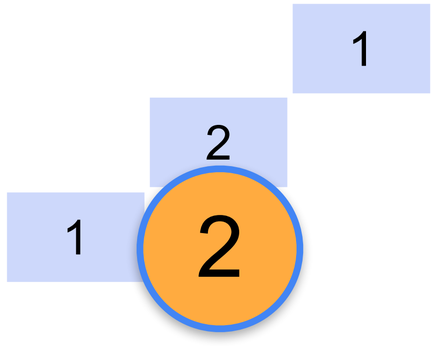
$+ \quad 1 \cdot 1 \cdot 1$



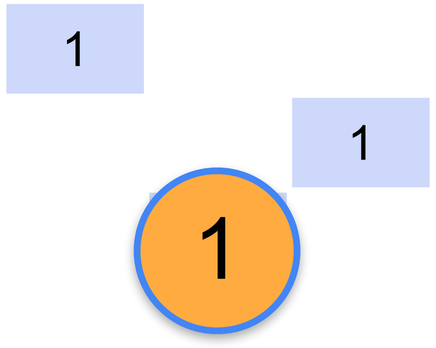
$+ \quad 1 \cdot 1 \cdot 1$

Det = 4+1+1
-2-1-2

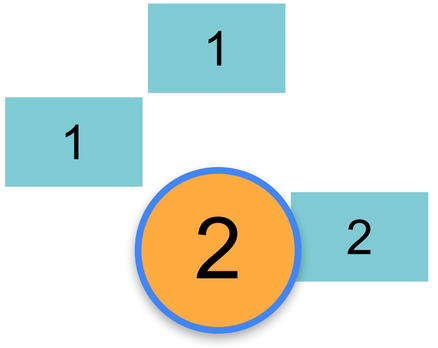
= 1



$- \quad 1 \cdot 2 \cdot 1$



$- \quad 1 \cdot 1 \cdot 1$



$- \quad 1 \cdot 1 \cdot 2$

Quiz: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

Solution: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

Determinant = 0

Singular

1	1	1
1	1	2
0	0	-1

Determinant = 0

Singular

1	1	1
0	2	2
0	0	3

Determinant = 6

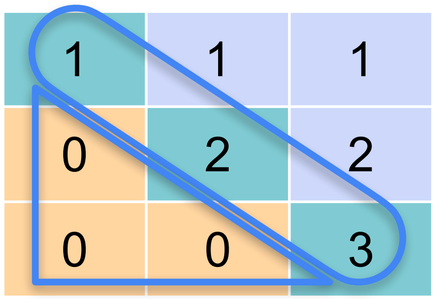
Non-singular

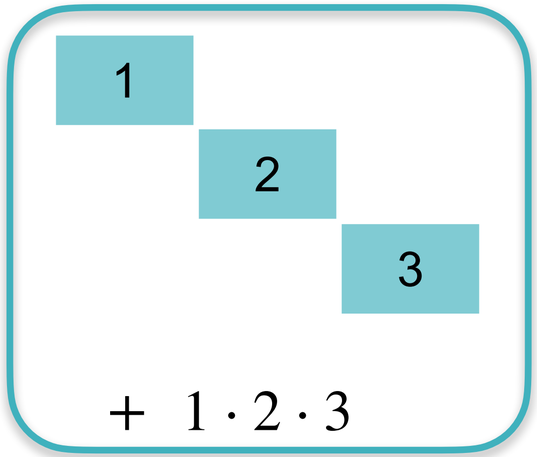
1	2	5
0	3	-2
2	4	10

Determinant = 0

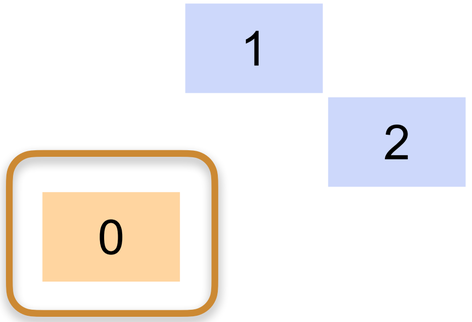
Singular

The determinant

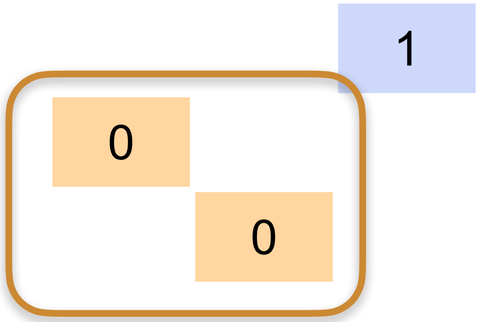




$+ \ 1 \cdot 2 \cdot 3$



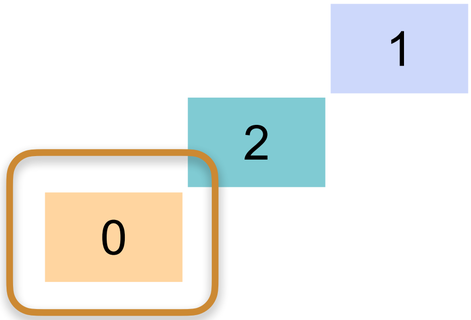
$+ \ 1 \cdot 2 \cdot 0$



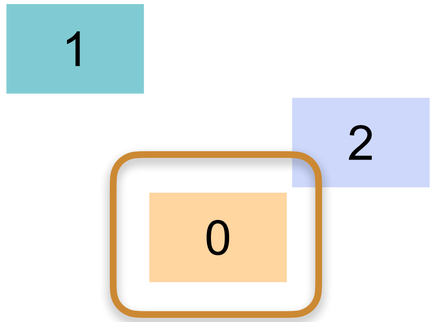
$+ \ 1 \cdot 0 \cdot 0$

Det = 6+0+0-0-0-0

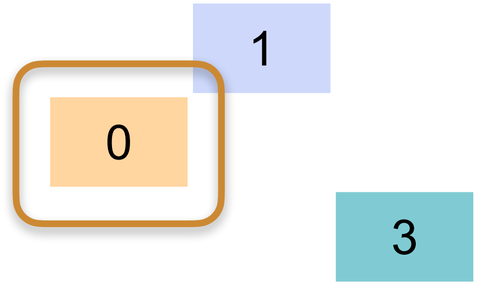
= 6



$- \ 1 \cdot 2 \cdot 0$

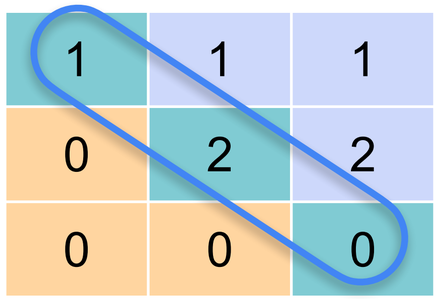


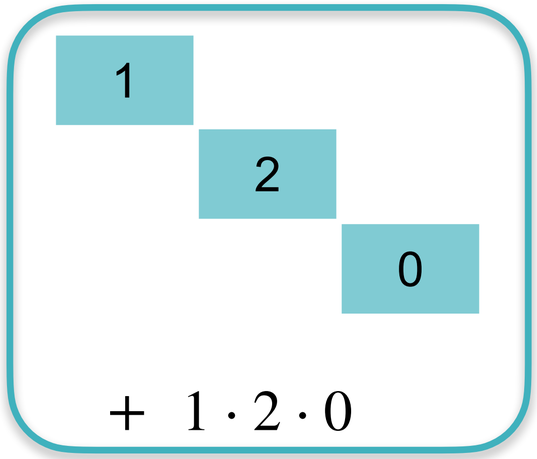
$- \ 1 \cdot 2 \cdot 0$



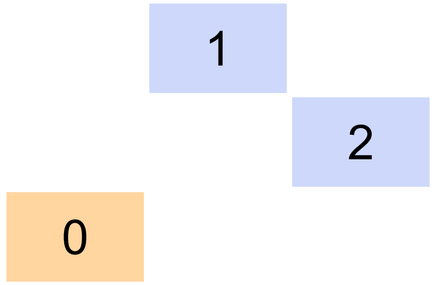
$- \ 1 \cdot 0 \cdot 3$

The determinant

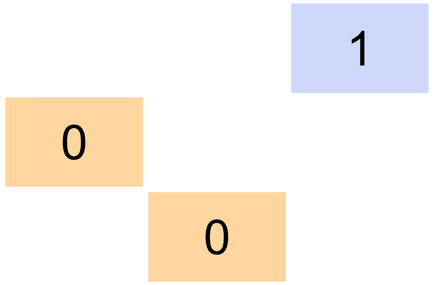




$+ \ 1 \cdot 2 \cdot 0$



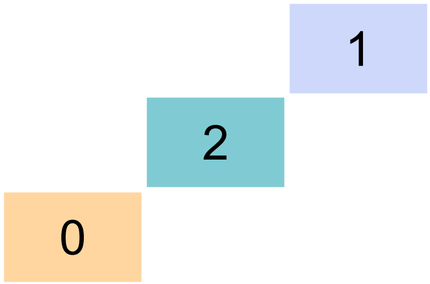
$+ \ 1 \cdot 2 \cdot 0$



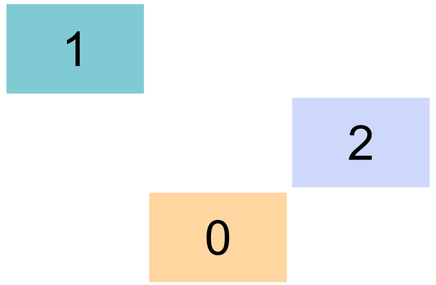
$+ \ 1 \cdot 0 \cdot 0$

Det = 0+0+0-0-0-0

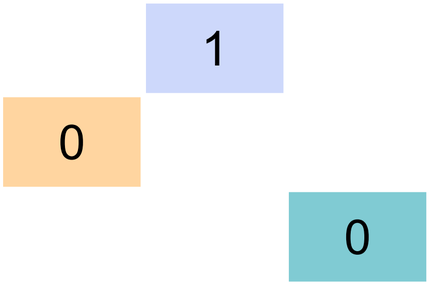
= 0



$- \ 1 \cdot 2 \cdot 0$



$- \ 1 \cdot 2 \cdot 0$



$- \ 1 \cdot 0 \cdot 0$

DeepLearning.AI

System of Linear Equations

Conclusion